Mathematics

Question1

If $f(x) = \tan\left(\frac{\pi}{\sqrt{x+1}+4}\right)$ is a real valued function, then the range of f is

Options:

A.

$[-1, 1]$

B.

$(0, 1]$

C.

$[-1, \infty)$

D.

R

Answer: B

Solution:

Given, $f(x) = \tan\left(\frac{\pi}{\sqrt{x+1}+4}\right)$

For $\sqrt{x+1}$ to be defined,

we have $x + 1 \geq 0 \Rightarrow x \geq -1$

$$\begin{aligned}
\therefore \quad & \sqrt{x+1} + 4 \geq 0 + 4 = 4 \\
\Rightarrow \quad & 0 < \frac{1}{\sqrt{x+1}+4} \leq \frac{1}{4} \\
\Rightarrow \quad & 0 < \frac{\pi}{\sqrt{x+1}+4} \leq \frac{\pi}{4} \\
\therefore \quad & f(x) = \tan\left(\frac{\pi}{\sqrt{x+1}} + 4\right), \text{ where} \\
& 0 < \frac{\pi}{\sqrt{x+1}+4} \leq \frac{\pi}{4}
\end{aligned}$$

So, the function $f(x)$ is strictly increasing in the interval $\left(0, \frac{\pi}{4}\right]$

And the range of the given function is $\left(\tan(0), \tan\left(\frac{\pi}{4}\right)\right]$, which is $(0, 1]$

$\therefore$ The range of $f(x)$ is $(0, 1]$

Question2

The range of the real value function $f(x) = \sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$ is

Options:

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

D. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

Answer: D

Solution:

Given, $f(x) = \sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$

The domain of $\sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$ is $[-1, 1]$

$\therefore \quad -1 \leq \sqrt{x^2 + x + 1} \leq 1$

Since $\sqrt{x^2 + x + 1}$ is always,

non-negative, so $0 \leq \sqrt{x^2 + x + 1} \leq 1$

Let $g(x) = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

The minimum value of $g(x)$ is $3/4$, which occurs at $x = -\frac{1}{2}$

$\therefore \quad x^2 + x + 1 \geq \frac{3}{4}$

$\Rightarrow \quad \sqrt{x^2 + x + 1} \geq \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} \leq \sqrt{x^2 + x + 1} \leq 1$

$\Rightarrow \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \leq \sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$

$\Rightarrow \quad \frac{\pi}{3} \leq f(x) \leq \frac{\pi}{2} \quad \leq \sin^{-1}$

Thus, the range of $f(x)=\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is $\left[\frac{\pi}{3},\frac{\pi}{2}\right]$

Question3

$1+(1+3)+(1+3+5)+(1+3+5+7)+\ldots$ to 10 terms $=$

Options:

A.

385

B.

285

C.

506

D.

406

Answer: A

Solution:

Given series

$1+(1+3)+(1+3+5)+(1+3+5+7)+\ldots$

to 10 terms.

The nth term of the series is the sum of first n odd numbers.

$$\begin{aligned} So,\ T_n &= 1+3+5+\ldots+(2n-1)=n^2 \\ \therefore\quad T_1 &= 1^2=1 \\ T_2 &= 2^2=4 \\ T_3 &= 3^2=9 \\ &\vdots \\ T_{10} &= 10^2=100 \end{aligned}$$

Thus, $S_{10}=\sum_{n=1}^{10} n^2$

We know that the sum of the first k squares is

$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}$

For $k = 10$

$$\begin{aligned} S_{10} &= \frac{10(10+1)(2\times 10+1)}{6} \\ &= \frac{10\times 11\times 21}{6} = \frac{2310}{6} = 385 \end{aligned}$$

$\therefore S_{10} = 385$

Question4

If the augmented matrix corresponding to the system of equations $x+y-z=1, 2x+4y-z=0$ and $3x+4y+5z=18$ is transformed to $\begin{bmatrix} 1 & a & 0 & -1 \\ 0 & 2 & 1 & b \\ 0 & 0 & c & 32 \end{bmatrix}$ then $\sqrt{a+b+c}=$

Options:

A.

1

B.

4

C.

9

D.

16

Answer: B

Solution:

Given equations are $x+y-z=1$,

$2x+4y-z=0$ and $3x+4y+5z=18$

The augmented matrix for the system is

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 2 & 4 & -1 & 0\\ 3 & 4 & 5 & 18\end{bmatrix}$$

We need to transform this matrix into

$$B=\begin{bmatrix}1 & a & 0 & -1\\ 0 & 2 & 1 & b\\ 0 & 0 & c & 32\end{bmatrix}$$

Now, perform the row operation for matrix A, we get

$R_2 \rightarrow R_2-2R_1$ and $R_3 \rightarrow R_3-3R_1$

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 0 & 2 & 1 & -2\\ 0 & 1 & 8 & 15\end{bmatrix}$$

Now, $R_3 \rightarrow R_3-\frac{1}{2}R_2$

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 0 & 2 & 1 & -2\\ 0 & 0 & 7.5 & 16\end{bmatrix}$$

Apply $R_3 \rightarrow 2R_3$, we get

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 0 & 2 & 1 & -2\\ 0 & 0 & 15 & 32\end{bmatrix}$$

Again apply $R_1 \rightarrow R_1+R_2$, we get

$$A=\begin{bmatrix}1 & 3 & 0 & -1\\ 0 & 2 & 1 & -2\\ 0 & 0 & 15 & 32\end{bmatrix}$$

Thus, $a=3, b=-2$ and $c=15$

$\Rightarrow \quad c=15$

$\therefore \quad \sqrt{a+b+c}=\sqrt{3+(-2)+15}$

$\quad =\sqrt{1+15}=\sqrt{16}=4$

Question5

If $\begin{vmatrix}9 & 25 & 16\\ 16 & 36 & 25\\ 25 & 49 & 36\end{vmatrix}=K$**, then** $K, K+1$ **are the roots of the equation**

Options:

A.

$x^2 - 13x + 42 = 0$

B.

$x^2 - 15x + 56 = 0$

C.

$x^2 - 19x + 90 = 0$

D.

$x^2 - 17x + 72 = 0$

Answer: D

Solution:

Given, $K = \begin{vmatrix} 9 & 25 & 16 \\ 16 & 36 & 25 \\ 25 & 49 & 36 \end{vmatrix}$

$$\begin{aligned} &= 9\begin{vmatrix} 36 & 25 \\ 49 & 36 \end{vmatrix} - 25\begin{vmatrix} 16 & 25 \\ 25 & 36 \end{vmatrix} + 16\begin{vmatrix} 16 & 36 \\ 25 & 49 \end{vmatrix} \\ &= 9(1296 - 1225) - 25(576 - 625) + 16(784 - 900) \\ &= 9(71) - 25(-49) + 16(-116) \\ &= 639 + 1225 - 1856 = 8 \\ &\therefore K = 8 \end{aligned}$$

So, the roots are K and $K + 1$, i.e., 8 and 9

$$\begin{aligned} \text{Sum of roots } &= 8 + 9 = 17 \\ &= \frac{-b}{a} = \alpha + \beta \end{aligned}$$

And product of roots $= 8 \times 9 = 72$

$= \frac{c}{a} = \alpha\beta$

So, the general form of a quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow \quad x^2 - 17x + 72 = 0$

Question6

$$A=\begin{bmatrix}1 & -3 & -5\\ -2 & 4 & -6\\ 7 & -11 & 13\end{bmatrix}, \textbf{ then } \sqrt{|\text{adj } A|}=$$

Options:

A.

64

B.

16

C.

36

D.

216

Answer: A

Solution:

Given, $A=\begin{bmatrix}1 & -3 & -5\\ -2 & 4 & -6\\ 7 & -11 & 13\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -3 & -5\\ -2 & 4 & -6\\ 7 & -11 & 13\end{vmatrix}$$
$$=1(4\times 13-(-6)\times(-11))-(-3)(-2\times 13-(-6)\times 7)-5(-2\times(-11)-7\times 4)$$
$$=1(52-66)+3(-26+42)-5(22-28)$$
$$=-14+48+30=64$$

Since, $|\text{adj } A|=|A|^{n-1}$, for $n\times n$ matrix A

So, for 3×3 matrix,

$$|\text{adj } A|=|A|^{3-1}=|A|^2$$
$$\therefore \quad \therefore \quad |\text{adj } A|=|A|^2=(64)^2=4096$$
$$\sqrt{|\text{adj } A|}=\sqrt{4096}=64$$

Question7

If $\Delta_r = \begin{vmatrix} \frac{1}{3r-2} & \frac{2}{3r-5} \\ 0 & \frac{3}{3r+1} \end{vmatrix}$ then $\sum_{r=1}^{33} \Delta_r =$

Options:

A.

0.99

B.

0.33

C.

0.66

D.

0.55

Answer: A

Solution:

$$\begin{aligned} \Delta r &= \begin{vmatrix} \frac{1}{3r-2} & \frac{2}{3r-5} \\ 0 & \frac{3}{3r+1} \end{vmatrix} \\ &= \frac{1}{(3r-2)} \times \frac{3}{(3r+1)} - \frac{2}{3r-5} \times 0 \\ &= \frac{3}{(3r-2)(3r+1)} \end{aligned}$$

Using partial fractions

$$\begin{aligned} \Delta r &= \frac{A}{3r-2} + \frac{B}{3r+1} \\ \Rightarrow \quad 3 &= A(3r+1) + B(3r-2) \end{aligned}$$

If $\quad r = \frac{2}{3}$, then

$$\begin{aligned} &\Rightarrow \quad 3 = A\left(3 \times \frac{2}{3} + 1\right) + B\left(3 \times \frac{2}{3} - 2\right) \\ &\Rightarrow 3A = 3 \\ &\Rightarrow \quad A = 1 \end{aligned}$$

If $\quad r = -\frac{1}{3}$, then

$$3 = B\left(3 \times \left(-\frac{1}{3}\right) - 2\right) = B(-1-2)$$

$$\Rightarrow \quad B = -1$$

So, $\Delta r = \dfrac{1}{3r-2} - \dfrac{1}{3r+1}$

$$\therefore \sum_{r=1}^{33} \Delta r = \sum_{r=1}^{33} \left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$$

$$\Rightarrow \left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) +$$

$$\ldots + \left(\frac{1}{97} - \frac{1}{100}\right)$$

$$\Rightarrow 1 - \frac{1}{100} = \frac{99}{100} = 0.99$$

Question8

If $\frac{2+3i}{i-2} - \frac{4i-3}{3+4i} = x + iy$, then $3x + y =$

Options:

A.

4

B.

-4

C.

-2

D.

2

Answer: B

Solution:

Given, $\frac{2+3i}{i-2} - \frac{4i-3}{3+4i} = x + iy$

Now, $\frac{2+3i}{i-2} = \frac{2+3i}{-2+i} \times \frac{(-2-i)}{(-2-i)}$

$\Rightarrow \quad \frac{-4-2i-6i-3i^2}{(-2)^2-i^2} = \frac{-1-8i}{5}$

And, $\frac{4i-3}{3+4i} \times \frac{3-4i}{3-4i}$

$\Rightarrow \quad \frac{-9+12i+12i-16i^2}{3^2-(4i)^2} = \frac{7+24i}{25}$

So, $\quad \frac{2+3i}{i-2} - \frac{4i-3}{3+4i}$

$$= \frac{-1}{5} - \frac{8i}{5} - \frac{7}{25} - \frac{24}{25}i$$
$$= \frac{-12}{25} - \frac{64}{25}i$$

On comparing, we get

$x = \frac{-12}{25}, y = \frac{-64}{25}$

So, $3x + y = 3\left(\frac{-12}{25}\right) - \frac{64}{25}$

$= -\frac{36}{25} - \frac{64}{25} = \frac{-100}{25} = -4$

Question9

Let $z = x + iy$ and $P(x, y)$ be a point on the argand plane. If z satisfies the condition $\arg\left(\frac{z-3i}{z+2i}\right) = \frac{\pi}{4}$, then the locus of P is

Options:

A.

$x^2 + y^2 - y - 6 = 0, (x, y) \neq (0, -2)$

B.

$x^2 + y^2 - x - y - 6 = 0, (x, y) \neq (0, -2)$

C.

$x^2 + y^2 + 5x - y - 6 = 0, (x, y) \neq (0, -2)$

D.

$x^2 + y^2 + x - y - 6 = 0, (x, y) \neq (0, -2)$

Answer: C

Solution:

Given, $\arg\left(\frac{z-3i}{z+2i}\right) = \frac{\pi}{4}$

Let $z = x + iy$

So, $\frac{z-3i}{z+2i} = \frac{x+iy-3i}{x+iy+2i}$

$\Rightarrow \frac{x+i(y-3)}{x+i(y+2)} \times \frac{x-i(y+2)}{x-i(y+2)}$

$$\Rightarrow \frac{x^2 - ix(y+2) + ix(y-3) + (y-3)(y+2)}{x^2 + (y+2)^2}$$
$$\Rightarrow \frac{x^2 + y^2 - y - 6 - i5x}{x^2 + (y+2)^2}$$

Let the complex numbers be $A + iB$, where

$A = \frac{x^2+y^2-y-6}{x^2+(y+2)^2}$ and $B = \frac{-5x}{x^2+(y+2)^2}$

So, arg $\left(\frac{z-3i}{z+2i}\right) = \left(\frac{\pi}{4}\right)$

$$\Rightarrow \quad \tan\left(\frac{B}{A}\right) = \frac{\pi}{4}$$
$$\Rightarrow \quad \frac{-5x}{x^2 + y^2 - y - 6} = \tan^{-1}\left(\frac{\pi}{4}\right) = 1$$
$$\Rightarrow \quad x^2 + y^2 - y - 6 = -5x$$
$$\Rightarrow \quad x^2 + 5x + y^2 - y - 6 = 0, \text{ where}$$
$x \neq 0$ and $\quad y \neq -2$

Question10

If ω is a complex cube root of unity and $x = \omega^2 - \omega + 2$, then

Options:

A.

$x^2 - 4x + 7 = 0$

B.

$x^2 + 4x + 7 = 0$

C.

$x^2 - 2x + 4 = 0$

D.

$x^2 + 2x + 4 = 0$

Answer: A

Solution:

Given, $x = \omega^2 - \omega + 2$ where w is a complex cube root of unity.

We know that $1 + \omega + \omega^2 = 0$

$\Rightarrow \omega^2 = -1 - \omega$

So, $x = (-1 - \omega) - \omega + 2 = 1 - 2\omega$

$\Rightarrow \omega = \frac{1-x}{2}$

So, $1 + \omega + \omega^2 = 0$

$$\begin{aligned}
\Rightarrow \quad & 1 + \left(\frac{1-x}{2}\right) + \left(\frac{1-x}{2}\right)^2 = 0 \\
\Rightarrow \quad & 1 + \frac{(1-x)}{2} + \frac{(1-x)^2}{4} = 0 \\
\Rightarrow \quad & 4 + 2(1-x) + 1 - 2x + x^2 = 0 \\
\Rightarrow \quad & 4 + 2 - 2x + 1 - 2x + x^2 = 0 \\
\Rightarrow \quad & x^2 - 4x + 7 = 0
\end{aligned}$$

Question11

The product of all the values of $(\sqrt{3} - i)^{\frac{3}{7}}$ is

Options:

A.

8

B.

-8

C.

$8i$

D.

$-8i$

Answer: D

Solution:

Let $z = \sqrt{3} - i$

$|z| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = 2$

And $\quad \theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ or $\frac{11\pi}{6}$

So, $\quad z = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

Now, $z^{\frac{3}{7}} = (\sqrt{3} - i)^{\frac{3}{7}}$

First, $z^3 = (\sqrt{3} - i)^3$

$$= 2^3\left(\cos\left(\frac{-3\pi}{6}\right) + i\sin\left(\frac{-3\pi}{6}\right)\right)$$
$$= 8\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) \qquad \ldots(i)$$

Now, $z^{\frac{3}{7}} = (\sqrt{3} - i)^{\frac{3}{7}}$

$= (8)^{\frac{1}{7}}\left(\cos\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right) + i\sin\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)\right) \qquad \ldots(ii)$

for $k = 0, 1, \ldots, 6$

So, $z^3 = 8(0 - i) = -8i \quad$ [Using Eq. (i)]

Now, we know that the product of nt h root of a complex number A is given by $(-1)^{n-1}A$, if A is a real number.

Since, $n = 7$ is odd, the product of the 7 roots of A is A itself.

$\therefore$ The product of the 7th roots of A is A itself.

$\therefore \quad (\sqrt{3} - i)^{\frac{3}{7}} = z^3$

So, product $= -8i$

Question12

α, β are the roots of the equation $\sin^2 x + b\sin x + c = 0$. If $\alpha + \beta = \frac{\pi}{2}$, then $b^2 - 1 =$

Options:

A. C

B. $2c$

C. C^2

D. $4c^2$

Answer: B

Solution:

Given, $\sin^2 x + b\sin x + c = 0$ and

$\alpha + \beta = \frac{\pi}{2}$

Let $y = \sin x$, thus we get

$y^2 + by + c = 0$

The roots of this equation are $\sin\alpha$ and $\sin\beta$.

So, sum of the roots $= \sin\alpha + \sin\beta = -\frac{b}{1}$

Product of the roots $= \sin\alpha \cdot \sin\beta = c$

Since, $\alpha + \beta = \frac{\pi}{2}$

$\Rightarrow \quad \beta = \frac{\pi}{2} - \alpha$

So, $\sin\alpha + \sin\left(\frac{\pi}{2} - \alpha\right) = -b$

$\Rightarrow \sin\alpha + \cos\alpha = -b$

$$\left[\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right]$$

$\Rightarrow (\sin\alpha + \cos\alpha)^2 = (-b)^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = (-b)^2$

$\Rightarrow 1 + 2c = b^2\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\Rightarrow b^2 - 1 = 2c$

Question13

The number of integral values of ' a ' for which the quadratic equation $ax^2 + ax + 5 = 0$ cannot have real roots is

Options:

A.

infinite

B.

20

C.

19

D.

5

Answer: C

Solution:

Given, quadratic equation is

$ax^2 + ax + 5 = 0$

For a quadratic equation

$Ax^2 + Bx + C = 0$ to have no real roots,

$$\Delta = B^2 - 4AC < 0$$

So, $\quad \Delta = a^2 - 4(a)(5) < 0$

$$\Rightarrow \quad a^2 - 20a < 0$$
$$\Rightarrow \quad a(a - 20) < 0$$

So, $\quad 0

Thus, a must be an integer.

So, the integral values of a in this range are $1, 2, 3, \ldots, 19$.

The number of such value is

$19 - 1 + 1 = 19$

Question14

If the roots of the equation $32x^3 - 48x^2 + 22x - 3 = 0$ are in arithmetic progression, then the square of the common difference of the roots is

Options:

A.

$\frac{1}{4}$

B.

$\frac{1}{16}$

C.

$\frac{1}{9}$

D.

$\frac{1}{25}$

Answer: B

Solution:

Given equation is

$32x^3 - 48x^2 + 22x - 3 = 0$

Since, the roots of this equation are in AP .

So, sum of roots $= a - d + a + a + d = 3a$

$= \frac{-(-48)}{32} = \frac{3}{2} \Rightarrow a = \frac{1}{2}$

Now, sum of the products of roots $= \frac{22}{32}$

$\Rightarrow (a-d)a + a(a+d) + (a-d)(a+d) = \frac{11}{16}$

$$\Rightarrow a^2 - ad + a^2 + ad + a^2 - d^2 = \frac{11}{16}$$

$$\Rightarrow 3a^2 - d^2 = \frac{11}{16} \Rightarrow 3\left(\frac{1}{2}\right)^2 - d^2 = \frac{11}{16}$$

$$\Rightarrow d^2 = \frac{3}{4} - \frac{11}{16} = \frac{12-11}{16} = \frac{1}{16}$$

$$\Rightarrow d^2 = \frac{1}{16}$$

So, the square of the common difference of the root is $\frac{1}{16}$.

Question15

If the sum of two roots of the equation $x^4 - 2x^3 + x^2 + 4x - 6 = 0$ is zero, then the sum of the squares of the other two roots is

Options:

A.

-6

B.

1

C.

-2

D.

0

Answer: C

Solution:

Given, equation is

$x^4 - 2x^3 + x^2 + 4x - 6 = 0$

Let the roots of this equation be α, β, γ and δ.

Sum of the roots

$= \alpha + \beta + \gamma + \delta = \frac{-(-2)}{1} = 2 \qquad \ldots (i)$

Sum of the products of roots

$$= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$$
$$= \frac{1}{1} = 1$$

Sum of products of roots taken three at a time

$$= \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$$
$$= \frac{-4}{1} = -4$$

Product of roots $= \alpha\beta\gamma\delta = \frac{-6}{1} = -6$

Since, the sum of two roots be zero, i.e.,

$$\alpha + \beta = 0$$
$$\Rightarrow \quad \beta = -\alpha$$

So, $\gamma + \delta = 2$ [from Eq. (i)]

And,

$$\alpha(-\alpha)+\alpha\gamma+\alpha\delta+(-\alpha\gamma)+(-\alpha)\delta+\gamma\delta=1$$
$$\Rightarrow \quad -\alpha^2+\alpha\gamma+\alpha\delta-\alpha\gamma-\alpha\delta+\gamma\delta=1$$
$$\Rightarrow \quad -\alpha^2+\gamma\delta=1 \qquad \ldots(ii)$$

And $\quad \alpha(-\alpha)\gamma\delta=-6$

$$\Rightarrow \quad -\alpha^2\gamma\delta=-6 \qquad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$\gamma\delta=1+\alpha^2$$
$$\Rightarrow \quad \alpha^2\left(1+\alpha^2\right)=6$$
$$\Rightarrow \quad \alpha^4+\alpha^2=6$$

Let $y=\alpha^2$, then we get

$$y^2+y-6=0$$
$$\Rightarrow \quad (y+3)(y-2)=0$$
$$\Rightarrow \quad y=-3 \text{ and } y=2$$

Since, α^2 must be non-negative,

$y=\alpha^2=2$

Then, $\gamma\delta=1+\alpha^2=1+2=3$

Now, $(\gamma+\delta)^2=\gamma^2+\delta^2+2\gamma\delta$

$$\Rightarrow \quad \gamma^2+\delta^2=(\gamma+\delta)^2-2\gamma\delta$$
$$=2^2-2(3)=4-6=-2$$

Question16

A student has to answer a multiple-choice question having 5 alternatives in which two or more than two alternatives are correct. Then, the number of ways in which the student can answer that question is

Options:

A.

31

B.

30

C.

27

D.

26

Answer: D

Solution:

The total number of ways to choose any subset of the 5 options $= 2^5 = 32$

These 32 subsets include choosing 2, 3, 4 or 5 options.

But choosing options 0 and 1 are not allowed.

Now, subsets with 0 options

$= {}^5C_0 = \frac{5!}{0!5!} = 1$

Subsets with 1 options

$= {}^5C_1 = \frac{5!}{1!4!} = 5$

So, total subsets with options 0 and 1

$= 1 + 5 = 6$

Thus, the number of ways in which the student can answer that questions

$= 32 - 6 = 26$

Question17

Number of triangles whose vertices are the points (x, y) in the XY-plane with integer coordinates satisfying $0 \leq x \leq 4$ and $0 \leq y \leq 4$ is

Options:

A.

2300

B.

2260

C.

2160

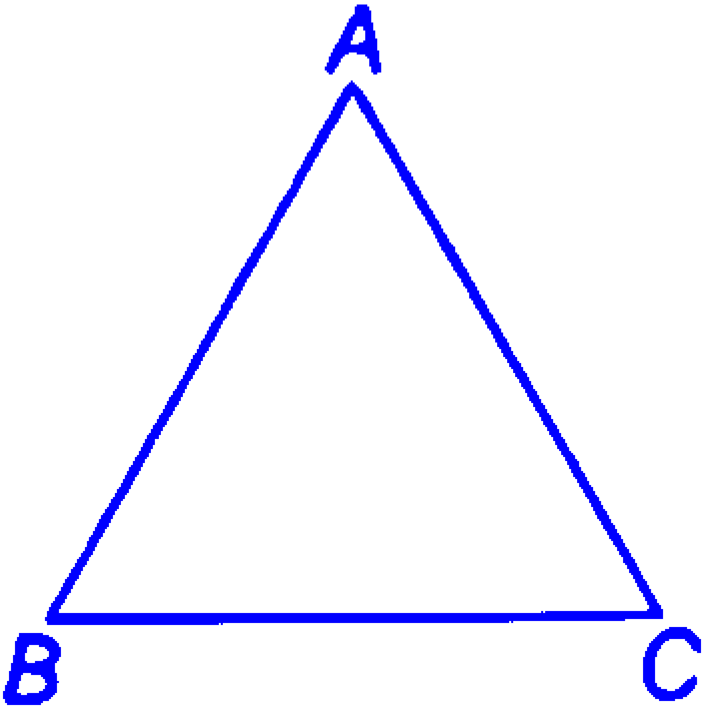
D.

2230

Answer: C

Solution:

Let the triangle be ABC with point (x, y),



where $0 \leq x \leq 4$ and $0 \leq y \leq 4$

So, the points (x, y) are $(0,0),(0,1),(0,2),(0,3),(0,4),(1,0),(1,1),(1,2),$ $(1,3),(1,4),(2,0),(2,1),(2,2),(2,3),(2,4),(3,0),(3,1),(3,2),(3,3),(3,4),$ $(4,0),(4,1),(4,2),(4,3),(4,4)$.

Total coordinates are 25 , but we need to select only 3 .

Now, the number of triangles with sets of 3 non-collinear points. Total number of ways to choose 3 points $= {}^{25}C_3$

But, we must subtract the number of collinear triplets.

Case I Horizontal lines

Number of ways to choose 3 collinear points

$\Rightarrow \quad {}^5C_3 = 10$

So, in 5 rows $= 5 \times 10 = 50$

Case II Vertical lines

Same as above $\rightarrow$ 5 columns

$= 5 \times 10 = 50$

Now, diagonal from bottom-left to top-right

$=$ Length 3 : 2 such diagonals + Length

4 : 2 diagonals + Length 5:1 diagonal

$$=2\left({}^{3}C_{3}\right)+2\left({}^{4}C_{3}\right)+{}^{5}C_{3}$$
$$2(1)+2(4)+10$$
$$=2+8+10=20$$

And, diagonals from top-left to bottom-right $= 20$

Total collinear triplets

$= 50 + 50 + 20 + 20 = 140$

So, total number of triangles $= {}^{25}C_3 - 140$

$\Rightarrow \quad 2300 - 140 = 2160$

Question18

If all the letters of the word 'HANDLE' are permuted in all possible ways and the words (with or without meaning) thus formed are arranged in dictionary order, then the rank of the word 'HELAND' is

Options:

A.

420

B.

422

C.

456

D.

475

Answer: B

Solution:

Total Alphabets are A, D, E, H, L, N Letters before H are A, D, E.

So, the number of letters before H $= 3 \times 5$!

$= 3 \times 120 = 360$

Letters before E are A, D.

So, the number of letters before $E = 2 \times 4$!

$= 2 \times 24 = 48$

Letters before L are A, D.

So, the number of letters before L

$= 2 \times 3! = 12$

Letters before N is D

So, the number of letters with N only

$\Rightarrow \quad 1 \times 1! = 1$

So, total words $= 360 + 48 + 12 + 1 = 421$

Now, only 1 letter remaining, that is D .

So, Rank of HELAND $= 421 + 1 = 422$

Question19

If the coefficient of 3rd term from the beginning in the expansion of $\left(ax^2 - \frac{8}{bx}\right)^9$ is equal to the coefficient of 3rd term from the end in the expansion of $\left(ax - \frac{2}{bx^2}\right)^9$, then the relation between a and b is

Options:

A.

$ab = -1$

B.

$ab = 1$

C.

$a^5b^5 = -2$

D.

$a^5b^5 = 2$

Answer: C

Solution:

Given, coefficient of 3rd term from beginning in the expansion $\left(ax^2 - \frac{8}{bx}\right)^9$ = Coefficient of 3rd term from end in the expansion $\left(ax - \frac{2}{bx^2}\right)^9$

For $\left(ax^2 - \frac{8}{bx}\right)^9$

$$\begin{aligned} T_3 &= {}^9C_2\left(ax^2\right)^7\left(\frac{-8}{bx}\right)^2 \\ &= \frac{9!}{2!7!}a^7x^{14} \cdot \frac{64}{b^2x^2} \\ &= \frac{36 \cdot a^7}{b^2} \cdot x^{12} \cdot 64 \end{aligned}$$

So, its coefficient is $\frac{36 \cdot 64 a^7}{b^2}$

For $\left(ax - \frac{2}{bx^2}\right)^9$

$$\begin{aligned} T_8 &= {}^9C_7(ax)^2\left(\frac{-2}{bx^2}\right)^7 \\ &= \frac{9!}{2!7!}a^2x^2 \cdot \frac{-128}{b^7x^{14}} \end{aligned}$$

So, if coefficient is

$= -36 \times 128\frac{a^2}{b^7}$

Now, $36 \cdot \frac{64a^7}{b^2} = -36 \cdot 128\frac{a^2}{b^7}$

$$\begin{aligned} \Rightarrow \quad 64 \cdot a^5b^5 &= -128 \\ \Rightarrow \quad a^5b^5 &= -2 \end{aligned}$$

Question20

If the expression $5^{2n} - 48n + k$ is divisible by 24 for all $n \in N$, then the least positive integral value of k is

Options:

A.

47

B.

48

C.

24

D.

23

Answer: D

Solution:

Given, $5^{2n}-48n+k$ is divisible by 24

Let $\quad E(n)=5^{2n}-48n+k$

and $\quad E(n) \equiv 0(\bmod 24), \forall n \in N$

$\Rightarrow \quad 5^{2n}-48n+k \equiv 0(\bmod 24)$

$\Rightarrow \quad k=48n-5^{2n}(\bmod 24)$

For $n=1$,

$$\begin{aligned} k & =48(1)-5^{2 \times 1}(\bmod 24) \\ & =48-25(\bmod 24) \\ & =23(\bmod 24) \end{aligned}$$

For $n=2$

$k=48(2)-5^{1}(\bmod 24)$

$$\begin{aligned} & =96-625(\bmod 24) \\ & =529(\bmod 24)=23(\bmod 24) \end{aligned}$$

Thus, divisibility for 24 to hold for all n, we must have $k \equiv 23(\bmod 24)$

Thus, the least positive integral value of k is 23 .

Question21

If $\frac{x^{3}+3}{(x-3)^{3}}=a+\frac{b}{x-3}+\frac{c}{(x-3)^{2}}+\frac{d}{(x-3)^{3}}$**, then** $(a+d)-(b+c)=$

Options:

A.

49

B.

15

C.

-30

D.

-5

Answer: D

Solution:

Given,

$\frac{x^3+3}{(x-3)^3} = a + \frac{b}{x-3} + \frac{c}{(x-3)^2} + \frac{d}{(x-3)^3} \qquad \ldots(i)$

Let $y = x - 3$

$\Rightarrow x = y + 3$

Then, $\frac{x^3+3}{(x-3)^3} = \frac{(y+3)^3+3}{y^3}$

$$= \frac{y^3 + 9y^2 + 27y + 27 + 3}{y^3}$$

$$= \frac{y^3 + 9y^2 + 27y + 30}{y^3}$$

$$= 1 + \frac{9}{y} + \frac{27}{y^2} + \frac{30}{y^3}$$

$$= 1 + \frac{9}{x-3} + \frac{27}{(x-3)^2} + \frac{30}{(x-3)^3}$$

Comparing this with Eq. (i), we get

$a = 1, b = 9, c = 27, d = 30$

So, $(a + d) - (b + c) = (1 + 30) - (9 + 27)$

$= 31 - 36 = -5$

Question22

If $\sin A = -\frac{60}{61}$, $\cot B = -\frac{40}{9}$ and neither A and B is in 4th quadrant, then $6\cot A + 4\sec B =$

Options:

A.

$\frac{26}{5}$

B.

$-\frac{26}{5}$

C.

-3

D.

3

Answer: C

Solution:

Given, $\sin A = -\frac{60}{61}$, $\cot B = -\frac{40}{9}$

Now, $\cos^2 A = 1 - \sin^2 A$

$$\Rightarrow \quad 1 - \left(-\frac{60}{61}\right)^2 \Rightarrow 1 - \frac{3600}{3721} = \frac{121}{3721}$$

$$\therefore \quad \cos A = \pm\frac{11}{61}$$

But $\sin A < 0$ and A and B are not in 4th quadrant.

So, A in third quadrant and B in 2nd quadrant

$\therefore \quad \cos A = -\frac{11}{61}$

And $\cot A = \frac{\cos A}{\sin A} = \frac{-11}{-60} = \frac{11}{60}$

$6\cot A = \frac{66}{60} = \frac{11}{10}$

Now, $\tan B = \frac{1}{\cot B} = \frac{-9}{40}$

And, $\sec^2 B = 1 + \tan^2 B$

$\Rightarrow \quad 1+\left(\frac{-9}{40}\right)^2=1+\frac{81}{1600}=\frac{41}{40}$

$\Rightarrow \quad \sec B=-\frac{41}{40}(\because B \text{ in 2nd quadrant })$

$\Rightarrow \quad 4 \sec B=\frac{-164}{40}=-\frac{41}{40}$

$\therefore \quad 6 \cot A+4 \sec B=\frac{11}{10}-\frac{41}{10}$

$\Rightarrow \quad \frac{-30}{10}=-3$

Question23

The period of the function $f(x)=\frac{2 \sin \left(\frac{\pi x}{3}\right) \cos \left(\frac{2 \pi x}{5}\right)}{3 \tan \left(\frac{7 \pi x}{2}\right)-5 \sec \left(\frac{5 \pi x}{3}\right)}$ is

Options:

A.

30

B.

60

C.

300

D.

150

Answer: A

Solution:

Given,

$f(x)=\frac{2 \sin \left(\frac{\pi x}{3}\right) \cos \left(\frac{2 \pi x}{5}\right)}{3 \tan \left(\frac{7 \pi x}{2}\right)-5 \sec \left(\frac{5 \pi x}{3}\right)}$

We know that

$$\sin(kx) \rightarrow \text{ period } = \frac{2\pi}{k}$$
$$\cos(kx) \rightarrow \text{ period } = \frac{2\pi}{k}$$
$$\tan(kx) \rightarrow \text{ period } = \frac{\pi}{k}$$
$$\sec(kx) \rightarrow \text{ period } = \frac{2\pi}{k}$$

So, for $\sin\left(\frac{\pi x}{3}\right)$, period $= \frac{2\pi}{\frac{\pi}{3}} = 6$

$$\cos\left(\frac{2\pi x}{5}\right) \text{ period } = \frac{2\pi}{\frac{2\pi}{5}} = 5$$
$$\tan\left(\frac{7\pi x}{2}\right) \text{ period } = \frac{\pi}{\frac{7\pi}{2}} = \frac{2}{7}$$
$$\sec\left(\frac{5\pi x}{3}\right) \text{ period } = \frac{2\pi}{\frac{5\pi}{3}} = \frac{6}{5}$$

Now, the LCM of $6, 5, \frac{2}{7}$ and $\frac{6}{5}$ is as follows

LCM of $\left\{\frac{6}{1}, \frac{5}{1}, \frac{2}{7}, \frac{6}{5}\right\} = \frac{\text{LCM of } (6,5,2,6)}{\text{HCF of } (1,1,7,5)}$

$\Rightarrow \frac{30}{1} = 30$

So, the period of $f(x)$ is 30 .

Question24

If $A + B + C = 4S$, then $\sin(2S - A)$

$+ \sin(2S - B) + \sin(2S - C) - \sin 2S =$

Options:

A.

$4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

B.

$4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

C.

$4\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

D.

$4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

Answer: D

Solution:

Given, $A+B+C=4S$

$\therefore A+B+C=4S=\pi \Rightarrow S=\frac{\pi}{4}$

$\therefore \sin(2S-A)+\sin(2S-B)+\sin(2S-C)-\sin 2S$

$=\sin\left(2\times\frac{\pi}{4}-A\right)+\sin\left(2\times\frac{\pi}{4}-B\right)+\sin\left(2\times\frac{\pi}{4}-C\right)-\sin\left(2\times\frac{\pi}{4}\right)$

$=\sin\left(\frac{\pi}{2}-A\right)+\sin\left(\frac{\pi}{2}-B\right)+\sin\left(\frac{\pi}{2}-C\right)-\sin\left(\frac{\pi}{2}\right)$

$=\cos A+\cos B+\cos C-1$

$=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Question25

The general solution of the equation $\sqrt{6-5\cos x+7\sin^2 x}-\cos x=0$ also satisfies the equation

Options:

A.

$\tan x+\cot x=2$

B.

$\cot x+\operatorname{cosec} x=1$

C.

$\tan x+\sec x=1$

D.

$\sec x+\operatorname{cosec} x=2$

Answer: C

Solution:

Given equation

$$\sqrt{6-5\cos x+7\sin^2 x}-\cos x=0$$
$$\Rightarrow \sqrt{6-5\cos x+7-7\cos^2 x}-\cos x=0$$
$$\Rightarrow \sqrt{13-5\cos x-7\cos^2 x}-\cos x=0$$

Squaring both sides, we get

$$\Rightarrow 13-5\cos x-7\cos^2 x=\cos^2 x$$
$$\Rightarrow 13-5\cos x-8\cos^2 x=0$$

$$\Rightarrow 8\cos^2 x+5\cos x+13=0$$

Let $y=\cos x$

$$\Rightarrow 8y^2+5y-13=0$$
$$y=\frac{-5\pm\sqrt{25-4(8)(-13)}}{2\times 8}$$
$$=\frac{-5\pm\sqrt{25+416}}{16}=\frac{-5\pm 21}{16}$$
$$\Rightarrow \quad y=\frac{-5+21}{16} \text{ or } y=\frac{-5-21}{16}$$
$$\Rightarrow \quad y=1 \text{ or } y=-\frac{26}{16}=-\frac{13}{8}(not\ possible)$$

$\therefore \quad \cos x=1 \Rightarrow \cos x=1$

And $\sin^2 x=1-\cos^2 x=0 \Rightarrow \sin x=0$

So, $\tan x=\frac{\sin x}{\cos x}=\frac{0}{1}=0$

$\sec x=\frac{1}{\cos x}=1$

Now, $\tan x+\cot x=0+\frac{1}{0}=\infty$

$$\cot x+\operatorname{cosec} x=\frac{1}{0}+\frac{1}{0}=\infty$$
$$\tan x+\sec x=0+1=1$$
$$\sec x+\operatorname{cosec} x=1+\frac{1}{0}=\infty$$

Question26

$$\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{6}{41}+\tan^{-1}\frac{9}{191}=$$

Options:

A.

$\tan^{-1}\frac{9}{10}$

B.

$\tan^{-1}\frac{18}{19}$

C.

$\tan^{-1}\frac{3}{191}$

D.

$\tan^{-1}\frac{6}{205}$

Answer: A

Solution:

$$\begin{aligned}
&\tan\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{6}{41}\right)\\
&\quad=\tan^{-1}\left(\frac{\frac{3}{5}+\frac{6}{41}}{1-\frac{3}{5}\cdot\frac{6}{41}}\right)=\tan^{-1}\left(\frac{\frac{153}{205}}{\frac{187}{205}}\right)\\
&\quad=\tan^{-1}\left(\frac{153}{187}\right)\\
&\text{Now, }\tan^{-1}\left(\frac{153}{187}\right)+\tan^{-1}\left(\frac{9}{191}\right)\\
&\quad=\tan^{-1}\left(\frac{\frac{153}{187}+\frac{9}{191}}{1-\frac{153}{187}\cdot\frac{9}{191}}\right)\\
&\quad=\tan^{-1}\left(\frac{\frac{29223+1683}{35717}}{1-\frac{1377}{35717}}\right)\\
&\quad=\tan^{-1}\left(\frac{30906}{34340}\right)=\tan^{-1}\left(\frac{9}{10}\right)
\end{aligned}$$

Question27

If $2\tanh^{-1}x=\sinh^{-1}\left(\frac{4}{3}\right)$, then $\cosh^{-1}\left(\frac{1}{x}\right)=$

Options:

A.

$\log(\sqrt{2}+1)$

B.

$\log(\sqrt{2}-1)$

C.

$\log(2+\sqrt{3})$

D.

$\log(2-\sqrt{3})$

Answer: C

Solution:

$$\text{Given, } 2\tanh^{-1}x = \sinh^{-1}\left(\frac{4}{3}\right)$$
$$\tanh^{-1}(x) = \tfrac{1}{2}\ln\left(\tfrac{1+x}{1-x}\right)$$
$$\Rightarrow \quad 2\tanh^{-1}(x) = \ln\left(\tfrac{1+x}{1-x}\right)$$
$$\text{So, } \ln\left(\tfrac{1+x}{1-x}\right) = \sinh^{-1}\left(\tfrac{4}{3}\right)$$
$$= \ln\left(\tfrac{4}{3} + \sqrt{\left(\tfrac{4}{3}\right)^2 + 1}\right)$$
$$\left[\because \sinh^{-1}(y) = \ln\left(y + \sqrt{y^2+1}\right)\right]$$
$$= \ln\left(\frac{4}{3} + \sqrt{\frac{16}{9} + 1}\right)$$
$$= \ln\left(\frac{4}{3} + \frac{5}{3}\right) \Rightarrow \ln\frac{9}{3} = \ln(3)$$
$$= \frac{1+x}{1-x} = 3 \Rightarrow 1 + x = 3 - 3x$$
$$= 4x = 2 \Rightarrow x = \frac{1}{2}$$
$$\text{Now, } \cosh^{-1}\left(\frac{1}{x}\right) = \cosh^{-1}(4$$
$$= \ln(2 + \sqrt{4-1})$$
$$\left[\because \cosh^{-1}(z) = \ln\left(z + \sqrt{z^2-1}\right)\right]$$
$$= \ln(2+\sqrt{3})$$

Question28

If p_1, p_2, p_3 are the altitudes and $a = 4, b = 5, c = 6$ are the sides of a $\triangle ABC$, then $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} =$

Options:

A.

$\frac{77}{225}$

B.

$\frac{44}{225}$

C.

$\frac{308}{225}$

D.

$\frac{22}{75}$

Answer: B

Solution:

Given, $a = 4, b = 5$ and $c = 6$

Since, p_1, p_2 and p_3 are altitudes of the sides a, b and c respectively.

$$\therefore \quad p = \frac{2\Delta}{\text{base}}$$
$$\Rightarrow \quad p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$$
$$\Rightarrow \quad \frac{1}{p_1^2} = \frac{a^2}{4\Delta^2}, \frac{1}{p_2^2} = \frac{b^2}{4\Delta^2}, \frac{1}{p_3^2} = \frac{c^2}{4\Delta^2}$$
$$\therefore \quad \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2}$$
$$= \frac{a^2}{4\Delta^2} + \frac{b^2}{4\Delta^2} + \frac{c^2}{4\Delta^2}$$
$$= \frac{a^2+b^2+c^2}{4\Delta^2}$$

We know that $s = \frac{a+b+c}{2}$

$= \frac{4+5+6}{2} = \frac{15}{2}$

So, $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

(Using heron's formula)

$$= \sqrt{\frac{15}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2}}$$
$$= \sqrt{\frac{1575}{16}} = \frac{\sqrt{1575}}{4}$$
$$= \Delta^2 = \frac{1575}{16}$$

And $a^2 + b^2 + c^2 = 4^2 + 5^2 + 6^2$

$= 16 + 25 + 36 = 77$

$$So,\ \frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{77}{4\Delta^2}$$
$$=\frac{77}{4\times\frac{1575}{16}}=\frac{77}{\frac{6300}{16}}$$
$$=\frac{77\times 16}{6300}=\frac{44}{225}$$

Question29

Let the angles A, B, C of a $\triangle ABC$ be in arithmetic progression. If the exradii r_1, r_2, r_3 of $\triangle ABC$ satisfy the condition $r_3^2=r_1r_2+r_2r_3+r_3r_1$, then $b=$

Options:

A.

$\frac{2a}{\sqrt{3}}$

B.

$\sqrt{2}a$

C.

$\sqrt{3}a$

D.

a

Answer: C

Solution:

Given angles A, B, C are in AP and $A+B+C=180^\circ \quad (\because$ angles of $\triangle ABC)$

So, $\frac{A+C}{2}=B$

$\therefore 2B+B=180^\circ$

$\Rightarrow \quad 3B=180^\circ$
$\Rightarrow \quad B=60^\circ$

Given that $r_3^2=r_1r_2+r_2r_3+r_3r_1$

Since, $B=60^\circ$, using the cosine rule for side b.

$$b^2 = a^2 + c^2 - 2ac\cos 60^\circ$$
$$= a^2 + c^2 - 2ac \cdot \frac{1}{2} = a^2 + c^2 - ac$$

We know that, $s^2 = r_1r_2 + r_2r_3 + r_3r_1$

And given, $r_3^2 = r_1r_2 + r_2r_3 + r_3r_1$

So, $r_3^2 = s^2$

$\Rightarrow r_3 = s$

Since, $r_3 = \frac{\Delta}{s-c}$

$$\Rightarrow \quad s = \frac{\Delta}{s-c} \Rightarrow \Delta = s(s-c)$$
$$\Rightarrow \quad \sqrt{s(s-a)(s-b)(s-c)} = s(s-c)$$

Squaring both sides, we get

$$\Rightarrow \quad s(s-a)(s-b)(s-c) = s^2(s-c)^2$$
$$\Rightarrow \quad (s-a)(s-b) = s(s-c)$$
$$\Rightarrow \quad s^2 - sa - sb + ab = s^2 - sc$$
$$\Rightarrow \quad ab = s(a+b-c)$$
$$\Rightarrow \quad ab = \frac{(a+b+c)}{2} \cdot (a+b-c)$$
$$\left[\because s = \frac{a+b+c}{2}\right]$$

$$\Rightarrow \quad 2ab = (a+b)^2 - c^2$$
$$\Rightarrow \quad 2ab = a^2 + b^2 + 2ab - c^2$$
$$\Rightarrow \quad a^2 + b^2 - c^2 = 0$$
$$\Rightarrow \quad c^2 = a^2 + b^2$$

So, the triangle is a right-angled triangle with right angle at c.

But, we have $B = 60^\circ$ and $C = 90^\circ$

So, $A = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Using the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\Rightarrow \quad \frac{a}{\sin 30^\circ} = \frac{b}{\sin 60^\circ}$$
$$\Rightarrow \quad \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} \Rightarrow 2a = \frac{2b}{\sqrt{3}}$$
$$\Rightarrow \quad \sqrt{3}a = b$$
$$\therefore \quad b = \sqrt{3}a$$

Question30

The position vectors of two points A and B are $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $7\hat{\mathbf{i}} - \hat{\mathbf{k}}$ respectively. The point P with position vector $-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ is on the line AB. If the point Q is the harmonic conjugate of P, then the sum of the scalar components of the position vector of Q is

Options:

A.

6

B.

4

C.

2

D.

0

Answer: A

Solution:

Given, $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{B} = 7\hat{\mathbf{i}} - \hat{\mathbf{k}}$,

$\mathbf{P} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

If P divides AB in the ratio $m : n$, then its harmonic conjugate Q divides AB in the ratio $-m : n$.

Now, $\mathbf{AB} = \mathbf{B} - \mathbf{A}$

$$= (7-1)\hat{\mathbf{i}} + (0-2)\hat{\mathbf{j}} + (-1-3)\hat{\mathbf{k}}$$
$$= 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

Let P divides AB in the ratio $m : n$, so

$\mathbf{P} = \frac{n\mathbf{A} + m\mathbf{B}}{m+n}$

$$\Rightarrow \quad -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}} = \frac{n(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + m(7\hat{\mathbf{i}} - \hat{\mathbf{k}})}{m+n}$$

$$\Rightarrow \quad (m+n)(-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = n(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + m(7\hat{\mathbf{i}} - \hat{\mathbf{k}})$$

Comparing the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ on both sides

$$\Rightarrow \quad -2(m+n) = n(1) + m(7)$$
$$\Rightarrow \quad -2m - 2n = n + 7m$$
$$\Rightarrow \quad -9m = 3n$$
$$\Rightarrow \quad n = -3m$$
$$\Rightarrow \quad \frac{m}{n} = \frac{1}{-3}$$
$$\therefore \quad \frac{-m}{n} = \frac{-1}{-3} = \frac{1}{3}$$

Now using section formula,

$$Q = \frac{3A + 1B}{3+1}$$
$$= \frac{3(\hat{i} + 2\hat{j} + 3\hat{k}) + (7\hat{i} - \hat{k})}{4}$$
$$= \frac{10\hat{i} + 6\hat{j} + 8\hat{k}}{4} = \frac{5}{2}\hat{i} + \frac{3}{2}\hat{j} + 2\hat{k}$$

So, sum of scalar components of

$$Q = \frac{5}{2} + \frac{3}{2} + 2$$
$$= \frac{8}{2} + 2 = 4 + 2 = 6$$

Question31

The point of intersection of the line joining the points $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and the plane passing through the points $\hat{\mathbf{i}}, 2\hat{\mathbf{j}}, 3\hat{\mathbf{k}}$ is

Options:

A.

$\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

B.

$\frac{1}{7}(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

C.

$\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

D.

$\frac{1}{7}(15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} - 9\hat{\mathbf{k}})$

Answer: D

Solution:

Let the points be $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and plane through points are $\mathbf{P}_1 = \hat{\mathbf{i}}, \mathbf{P}_2 = 2\hat{\mathbf{j}}, \mathbf{P}_3 = 3\hat{\mathbf{k}}$

Let the point on the line be

$$\begin{aligned}
&\mathbf{r}(t) = \mathbf{A} + t(\mathbf{B} - \mathbf{A})\\
&\Rightarrow \mathbf{r}(t) = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + t[(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})\\
&\qquad (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})]\\
&\quad = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + t(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})\\
&\quad = (1+t)\hat{\mathbf{i}} + (2-3)\hat{\mathbf{j}} + (1-2t)\hat{\mathbf{k}}
\end{aligned}$$

Now, vectors in the plane are

$$\begin{aligned}
\mathbf{v}_1 &= \mathbf{P}_2 - \mathbf{P}_1 = 2\hat{\mathbf{j}} - \hat{\mathbf{i}}\\
\mathbf{v}_2 &= \mathbf{P}_3 - \mathbf{P}_1 = 3\hat{\mathbf{k}} - \hat{\mathbf{i}}
\end{aligned}$$

Normal vectors, $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$

$$\Rightarrow \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (6-0)\hat{\mathbf{i}} - (-3-0)\hat{\mathbf{j}} + (0+2\hat{\mathbf{k}}$$

$$\Rightarrow \quad 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Using point $\mathbf{P}_1 = \hat{\mathbf{i}}$ to get plane equation is

$$\mathbf{n} \cdot (\mathbf{r} - \hat{\mathbf{i}}) = 0$$

$$\begin{aligned}
&\Rightarrow \quad \langle 6,3,2\rangle \cdot \langle x-1, y, z\rangle = 0\\
&\Rightarrow \quad 6x - 6 + 3y + 2z = 0\\
&\Rightarrow \quad 6x + 3y + 2z = 6 \qquad\qquad \ldots(i)
\end{aligned}$$

Put the value of $\mathbf{r}(t) = (1+t, 2-3t, 1-2t)$ into the plane Eq. (i), we get

$$\begin{aligned}
&6x + 3y + 2z = 6\\
&\Rightarrow 6(1+t) + 3(2-3t) + 2(1-2t) = 6\\
&\Rightarrow 6 + 6t + 6 - 9t + 2 - 4t = 6\\
&\Rightarrow -7t + 14 = 6 \Rightarrow 7t = 8\\
&\Rightarrow \quad t = \frac{8}{7}\\
&\text{So, } \mathbf{r}\left(\frac{8}{7}\right) = \left(1 + \frac{8}{7}\right)\hat{\mathbf{i}} + \left(2 - 3 \times \frac{8}{7}\right)\hat{\mathbf{j}} + \left(1 - 2 \times \frac{8}{7}\right)\hat{\mathbf{k}}\\
&\Rightarrow \frac{15}{7}\hat{\mathbf{i}} - \frac{10}{7}\hat{\mathbf{j}} - \frac{9}{7}\hat{\mathbf{k}}
\end{aligned}$$

Question32

If a and b are two vectors such that $|\mathbf{a}| = 5$, $|\mathbf{b}| = 12$ and $|\mathbf{a} - \mathbf{b}| = 13$, then $|2\mathbf{a} + \mathbf{b}| =$

Options:

A.

$2\sqrt{61}$

B.

15

C.

$61\sqrt{2}$

D.

17

Answer: A

Solution:

Given, $|\mathbf{a}| = 5$, $|\mathbf{b}| = 12|\mathbf{a} - \mathbf{b}| = 13$

Now, $|\mathbf{a} - \mathbf{b}|^2 = 13^2 = 169$

$$\begin{aligned}
\Rightarrow \quad & |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b} = 169 \\
\Rightarrow \quad & 5^2 + 12^2 - 2\mathbf{a} \cdot \mathbf{b} = 169 \\
\Rightarrow \quad & -2\mathbf{a} \cdot \mathbf{b} = 169 - 25 - 144 = 0 \\
\Rightarrow \quad & \mathbf{a} \cdot \mathbf{b} = 0
\end{aligned}$$

So, $\mathbf{a} \perp \mathbf{b}$

Now, $|2\mathbf{a} + \mathbf{b}|^2 = (2\mathbf{a} + \mathbf{b}) \cdot (2\mathbf{a} + \mathbf{b})$

$$\begin{aligned}
\Rightarrow \quad & 4\mathbf{a} \cdot \mathbf{a} + 4\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b} \\
& = 4|\mathbf{a}|^2 + 4\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 \\
& = 4(5)^2 + 4 \times 0 + (1)^2 \\
& = 100 + 0 + 144 = 244 \\
\therefore \quad & |\mathbf{2a} + \mathbf{b}| = \sqrt{244} = 2\sqrt{61}
\end{aligned}$$

Question33

If $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are two vectors, then $(\mathbf{a} + 2\mathbf{b}) \times (3\mathbf{a} - \mathbf{b})$

Options:

A.

$2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

B.

$6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

C.

$14\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

D.

$14\hat{\mathbf{i}} + 42\hat{\mathbf{j}} - 35\hat{\mathbf{k}}$

Answer: D

Solution:

Given, $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and

$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\mathbf{a} + 2\mathbf{b} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) + 2(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$\Rightarrow \quad (1+4)\hat{\mathbf{i}} + (-2+2)\hat{\mathbf{j}} + (-2+4)\hat{\mathbf{k}}$

$\Rightarrow \quad 5\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$3\mathbf{a} - \mathbf{b} = 3(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$\Rightarrow \quad (3-2)\hat{\mathbf{i}} + (-6-1)\hat{\mathbf{j}} + (-6-2)\hat{\mathbf{k}}$

$\Rightarrow \quad \hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$$\text{Now, } (\mathbf{a} + 2\mathbf{b}) \times (3\mathbf{a} - \mathbf{b}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 5 & 0 & 2 \\ 1 & -7 & -8 \end{vmatrix}$$

$\Rightarrow (0 - (-14))\hat{\mathbf{i}} - (-40 - 2)\hat{\mathbf{j}} + (-35 - 0)\hat{\mathbf{k}}$

$\Rightarrow 14\hat{\mathbf{i}} + 42\hat{\mathbf{j}} - 35\hat{\mathbf{k}}$

Question34

The shortest distance between the lines

$\mathbf{r} = (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + t(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ and
$\mathbf{r} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) + s(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ is

Options:

A.

7

B.

8

C.

9

D.

12

Answer: B

Solution:

Given,

$\mathbf{r}_1 = (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + t(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

So, $\mathbf{a}_1 = (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$,

$\mathbf{b}_1 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

And $\mathbf{r}_2 = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) + s(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

So, $\mathbf{a}_2 = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}), \mathbf{b}_2 = (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Since, shortest distance,

$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$

Now, $\mathbf{a}_2 - \mathbf{a}_1 = \langle 1, 2, -4 \rangle - \langle 3, -5, 2 \rangle$

$= \langle 1 - 3, 2 - (-5), -4 - 2 \rangle$

$$= < -2, 7, -6 >$$

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 3 & -1 \\ 6 & 3 & -2 \end{vmatrix}$$

$$= (-6 - (-3))\hat{\mathbf{i}} - (-8 - (-6))\hat{\mathbf{j}} + (12 - 18)\hat{\mathbf{k}}$$

$$= -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$$

$$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{(-3)^2 + 2^2 + (-6)^2}$$

$$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

$$\text{So,} \quad d = \frac{|\langle -2, 7, -6 \rangle \cdot \langle -3, 2, -6 \rangle|}{7}$$

$$= \frac{|6 + 14 + 36|}{7} = \frac{56}{7} = 8$$

Question35

The mean deviation from the median for the following data is

x_i	2	9	8	3	5	7
f_i	5	3	1	6	6	1

Options:

A.

2

B.

$\frac{8}{3}$

C.

$\frac{9}{2}$

D.

9

Answer: A

Solution:

Given data,

x_i	2	9	8	3	5	7
f_i	5	3	1	6	6	1

x_i	f_i	Cumulative frequency
2	5	5
9	3	8
8	1	9
3	6	15
5	6	21
7	1	22

Total frequency $N=22$ (even)

$\Rightarrow \quad \frac{N}{2}=\frac{22}{2}=11$

Since, the cumulative frequency just greater than $\frac{N}{2}$ i.e., 11 is 15 and the value of x corresponding to 15 is 3 .

$\therefore$ Median $=3$

Now, mean deviation

$=\frac{1}{N} \Sigma f_i\left|x_i-\text { Median }\right|$

x_i	f_i	$\|x_i-3\|$	$f_{\mid}\|x_i-3\|$
2	5	1	5
9	3	6	18
8	1	5	5
3	6	0	0
5	6	2	12
7	1	4	4

So, $\Sigma f_i\left|x_i-3\right|=44$

So, mean deviation $=\frac{44}{22}=2$

Question36

If three smallest squares are chosen at-random on a chess board, then the probability of getting them in such a way that they are all together in a row or in a column is

Options:

A.

$\frac{73}{5208}$

B.

$\frac{1}{434}$

C.

$\frac{96}{217}$

D.

$\frac{479}{504}$

Answer: B

Solution:

Total number of ways to choose any 3 distinct squares $= {}^{64}C_3$

$\Rightarrow \quad \frac{64!}{3!61!} = \frac{64\times63\times62}{6} = 41664$

Since, each row has 8 squares.

In each row, the number of ways to choose 3 adjacent squares $= 8 - 3 + 1 = 6$

So, total for 8 rows $= 8 \times 6 = 48$

Similarly, total for 8 columns $= 8 \times 6 = 48$

$\therefore$ Total favourable outcomes

$\Rightarrow \quad 48 + 48 = 96$

Hence, required probability

$\Rightarrow \quad \frac{96}{41664} = \frac{1}{434}$

Question37

If three cards are drawn randomly from a pack of 52 playing cards then the probability of getting exactly, one spade card, exactly one king and exactly one card having a prime number is

Options:

A.

$\frac{72}{221}$

B.

$\frac{72}{5525}$

C.

$\frac{16}{425}$

D.

$\frac{144}{5525}$

Answer: B

Solution:

Number of spade cards $= 13$, number of kings $= 4$

Prime numbered cards are $2, 3, 5$ and 7

So, total primes cards $= 4$ suits $\times 4$

numbers

$\Rightarrow$ 16 cards

But, a single card can be both a king and a spade or a prime and a spade.

So, the number of favourable outcomes.

Case I Spade that is not a king or prime $= 13 - 1 - 4 = 8$

Case II King that is not a spade or prime = 3

Case III Prime card that is not a spade or king $= 16 - 4 = 12$

So, total favourable cases

$\Rightarrow \quad 8 \times 3 \times 12 = 288$

and total number of ways to choose 3 cards from 52 cards $= {}^{52}C_3$

$= \frac{52!}{3!49!} = \frac{52 \times 51 \times 50}{6} = 22100$

Thus, required probability

$= \frac{288}{22100} = \frac{72}{5525}$

Question38

Urn A contains 6 white and 2 black balls; run B contains 5 white and 3 black balls and urn C contains 4 white and 4 black balls. if an urn is chosen at random and a ball is drawn at random from it, then the probability that the ball drawn is white is

Options:

A.

$\frac{3}{8}$

B.

$\frac{5}{8}$

C.

$\frac{1}{2}$

D.

$\frac{3}{4}$

Answer: B

Solution:

Since, there are three Urn.

So, $P(\text{Urn } A) = \frac{1}{3} = P(\text{Urn } B) = P(\text{Urn } C)$

Now, probability of drawing a white ball from each Urn

Urn A 6 white balls, 2 black balls.

Total = 8 balls

$P(\text{ White/Urn A}) = \frac{6}{8} = \frac{3}{4}$

Urn B 5 white balls, 3 black balls

Total = 8 balls

$P(\text{ White/Urn B }) = \frac{5}{8}$

Urn C 4 white balls, 4 black balls

Total = 8 balls

$P(\text{ White } / \text{ Urn C}) = \frac{4}{8} = \frac{1}{2}$

So, the total probability of drawing a white ball is

$P(\text{ White }) = P(\text{ White } / \text{ Urn A}) \cdot P(\text{ Urn A})$

$+P(\text{ White } / \text{ Urn B}) \cdot P(\text{ Urn B})$

$+P(\text{ White } / \text{ Urn C}) \cdot P(\text{ Urn C})$

$$= \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{5}{8} + \frac{1}{3} \times \frac{1}{2}$$
$$= \frac{1}{4} + \frac{5}{24} + \frac{1}{6}$$
$$= \frac{6+5+4}{24} = \frac{15}{24} = \frac{5}{8}$$

Question39

If three dice are thrown, then the mean of the sum of the numbers appearing on them is

Options:

A.

58.5

B.

76.66

C.

71.75

D.

10.5

Answer: D

Solution:

The numbers on a single die are 1 , 2, 3, 4, 5 and 6 .

So, the mean $= \frac{1+2+3+4+5+6}{6}$

$= \frac{21}{6} = 3.5$

When multiple dice are thrown, the mean of the sum of the numbers appearing on them = the sum of means of individual dice.

Since, each die has mean 3.5.

So, the mean of the sum of the numbers appearing on three dice

$= 3.5 + 3.5 + 3.5 = 10.5$

Question40

If $X \sim B(7, P)$ is a binomial variate and $P(X=3) = P(X=5)$, then $P =$

Options:

A.

$\frac{5-\sqrt{10}}{3}$

B.

$\frac{\sqrt{10}-2}{3}$

C.

$\frac{5-\sqrt{15}}{2}$

D.

$\frac{\sqrt{15}-3}{2}$

Answer: C

Solution:

Given $X \sim B(7, P)$ is a binomial variate and $P(X=3) = P(X=5)$

Using the binomial probability formula

$P(X=K) = \binom{n}{K} P^K (1-P)^{n-K}$

Here $n = 7$,

So, $P(X=3)=\binom{7}{3}P^3(1-P)^4$

And, $P(X=5)=\binom{7}{5}P^5(1-P)^2$

$$\therefore \binom{7}{3}P^3(1-P)^4=\binom{7}{5}P^5(1-P)^2$$
$$\Rightarrow 35P^3(1-P)^4=21P^5(1-P)^2$$
$$\Rightarrow 35(1-P)^2=21P^2,\ \text{where } P\neq 0, P\neq 1$$
$$\Rightarrow 5(1-P)^2=3P^2$$
$$\Rightarrow 5\left(1-2P+P^2\right)=3P^2$$
$$\Rightarrow 5-10P+5P^2-3P^2=0$$
$$\Rightarrow 2P^2-10P+5=0$$
$$\Rightarrow P=\frac{-(-10)\pm\sqrt{100-4\times2\times5}}{2\times2}$$
$$=\frac{10\pm\sqrt{100-40}}{4}$$
$$=\frac{10\pm2\sqrt{15}}{4}=\frac{5\pm\sqrt{15}}{2}$$

But $P=\frac{5+\sqrt{15}}{2}>1$, So it can't be a valid probability.

$\therefore P=\frac{5-\sqrt{15}}{2}$

Question41

If the points $A(2,3), B(3,2)$ form a triangle with a variable point $p\left(t,t^2\right)$, where t is a parameter, then the equation of the locus of the centroid of $\triangle ABC$ is

Options:

A.

$9x^2-30x-3y+20=0$

B.

$3x^2-10x-y+10=0$

C.

$9y^2-30y-3x+20=0$

D.

$3y^2-10y-x+10=0$

Answer: B

Solution:

Let the centroid be $G(x, y)$. The coordinates of the centroid of triangles with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are

$x = \frac{x_1+x_2+x_3}{3}$ and $y = \frac{y_1+y_2+y_3}{3}$

Substituting the coordinates of A, B and P.

$$x = \frac{2+3+t}{3} = \frac{5+t}{3},$$
$$y = \frac{3+2+t^2}{3} = \frac{5+t^2}{3}$$

$$\Rightarrow \quad 3x - 5 = t, y = \frac{5+(3x-5)^2}{3}$$
$$(\because t = 2x - 5)$$
$$\Rightarrow \quad 3y = 5 + 9x^2 + 25 - 30x$$
$$\Rightarrow \quad 9x^2 - 30x - 3y + 30 = 0$$
$$\Rightarrow \quad 3x^2 - 10x - y + 10 = 0$$

The equation of the locus of the centroid of $\triangle ABC$ is $3x^2 - 10x - y + 10 = 0$

Question42

If (h, k) is the new origin to be chosen to eliminate first degree terms from the equation $S \equiv 2x^2 - xy - y^2 - 3x + 3y = 0$ by translation and if θ is the angle with which the axes are to be rotated about the origin in anti-clockwise direction to eliminate xy-term from $S = 0$, then $\tan 2\theta =$

Options:

A.

$h + k$

B.

$h - k$

C.

hk

D.

$-\frac{h}{3k}$

Answer: D

Solution:

Let $S(x, y) = 2x^2 - xy - y^2 - 3x + 3y$

$\frac{\partial S}{\partial x} = 4x - y - 3, \frac{\partial S}{\partial y} = -x - 2y + 3$

Solve, $\frac{\partial S}{\partial x} = 0$ and $\frac{\partial S}{\partial y} = 0$, we get

$4x - y - 3 = 0$ and $-x - 2y + 3 = 0$

Solving these two equations for $x = h$ and $y = k$

$4h - k - 3 = 0$ and $-h - 2k + 3 = 0$
$\therefore h = 1, k = 1$

So, the new origin is $(h, k) = (1, 1)$

We know that a general second-degree equation

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation θ satisfies

$\tan 2\theta = \frac{B}{A-C}$

Here, $S \equiv 2x^2 - xy - y^2 - 3x + 3y = 0$, we have

$A = 2B = -1$ and $C = -1$
$\therefore \tan 2\theta = \dfrac{-1}{2-(-1)} = -\dfrac{1}{3}$

Now, $h + k = 1 + 1 = 2h - k = 1 - 1 = 0,$

$hk = 1 \times 1 = 1, \frac{h}{3k} = \frac{1}{3}$

So, $\tan 2\theta = \frac{-1}{3} = \frac{-h}{3k}$

Question43

A line L perpendicular to the line $5x - 12y + 6 = 0$ makes positive intercept on the Y-axis. If the distance from the origin to the line L is 2 units and the angle made by the perpendicular drawn from the origin to the line L with positive X-axis is θ, then $\tan\theta + \cot\theta =$

Options:

A.

$\frac{25}{12}$

B.

$\frac{625}{168}$

C.

$\frac{169}{60}$

D.

$\frac{1681}{360}$

Answer: C

Solution:

Give line is $5x-12y+6=0$

Slope of this line is $m_1=\frac{-5}{-12}=\frac{5}{12}$

Since, line L is perpendicular to this line, the product of their slope is -1 . Let the slope of line L be m_L.

Then, $m_L\times\frac{5}{12}=-1$

$\Rightarrow \quad m_L=-\frac{12}{5}$

The equation of a line in normal form is $x\cos\theta+y\sin\theta=P$, where P is the distance from the origin to the line and θ is the angle made by the perpendicular from the origin.

We have $P=2$ unit, So, the equation of line is $x\cos\theta+y\sin\theta=2$

Slope, $m_L=\frac{-\cos\theta}{\sin\theta}=-\cot\theta$

$\therefore -\cot\theta=\frac{-12}{5}\Rightarrow\cot\theta=\frac{12}{5}$

For y-intercept, $x=0$ then $y\sin\theta=2$

$\Rightarrow \quad y=\frac{2}{\sin\theta}$

And since y-intercept is positive, so

$$\frac{2}{\sin\theta}>0$$
$$\Rightarrow\sin\theta>0$$

$\Rightarrow\cot\theta=\frac{12}{5}>0, \theta$ must be in first quadrant.

Now, $\tan\theta=\frac{1}{\cot\theta}=\frac{5}{12}$

So, $\tan\theta+\cot\theta=\frac{5}{12}+\frac{12}{5}$

$= \frac{25+144}{60} = \frac{169}{60}$

Question44

If a line L passing through a point $A(2,3)$ intersects another line $4x - 3y - 19 = 0$ at the point B such that $AB = 4$, then the angle made by the line L with positive X-axis in anti-clockwise direction is

Options:

A.

$\tan^{-1}\left(-\frac{3}{4}\right)$

B.

$\tan^{-1}\left(\frac{3}{4}\right)$

C.

$\frac{\pi}{4}$

D.

$-\frac{\pi}{4}$

Answer: A

Solution:

Let the angle made by Line L with positive X-axis be θ.

So, $B = (Ax + AB\cos\theta, Ay + AB\sin\theta)$

Given, $A(2,3)$ and $AB = 4$, we have

$B = (2 + 4\cos\theta, 3 + 4\sin\theta)$

Since, point B lies on the line $4x - 3y - 19 = 0$, substitute the coordinate of B into this equation.

$$\begin{aligned}
&4(2+4\cos\theta) - 3(3+4\sin\theta) - 19 = 0 \\
\Rightarrow \quad & 8 + 16\cos\theta - 9 - 12\sin\theta - 19 = 0 \\
\Rightarrow \quad & 16\cos\theta - 12\sin\theta - 20 = 0 \\
\Rightarrow \quad & 4\cos\theta - 3\sin\theta - 5 = 0 \\
\Rightarrow \quad & \frac{4}{5}\cos\theta - \frac{3}{5}\sin\theta = 1
\end{aligned}$$

Let $\cos\alpha = \frac{4}{5}$ and $\sin\alpha = \frac{3}{5}$ for some angle α in the first quadrant.

$\therefore \cos\alpha \cdot \cos\theta - \sin\alpha \cdot \sin\theta = 1$

$\Rightarrow \quad \cos(\alpha + \theta) = 1$
$\Rightarrow \quad \alpha + \theta = 2n\pi, \text{ for some integer } n.$
$\Rightarrow \quad \alpha + \theta = 0$
$\Rightarrow \quad \alpha = -\theta \text{ or } \alpha + \theta = 2\pi$
$\Rightarrow \quad \theta = 2\pi - \alpha$

The angle made by the line L with positive X-axis in the anti-clockwise direction is taken as a positive angle, where

$\cos\theta = \frac{4}{5}$ and $\sin\theta = -\frac{3}{5}$

This angle lies in the fourth quadrant.

So, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-3}{4}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{-3}{4}\right) = -\tan^{-1}\left(\frac{3}{4}\right)$

Question45

A variable straight-line L with negative slope passes through the point $(4, 9)$ and cuts the positive coordinate axes in A and B. If O is the origin, then the minimum value of $OA + OB$ is

Options:

A.

25

B.

12

C.

13

D.

5

Answer: A

Solution:

Let the equation of the line L be $\frac{x}{a}+\frac{y}{b}=1$, where a and b are the x and y intercepts, respectively. So, $OA=a$ and $OB=b, a>0, b>0$.

Here, the line passes through the point $(4,9)$, So,

$\frac{4}{a}+\frac{9}{b}=1$

We want to minimize $a+b$. So consider

$$(a+b)\left(\frac{4}{a}+\frac{9}{b}\right)=4+\frac{9a}{b}+\frac{4b}{a}+9$$
$$=13+\frac{9a}{b}+\frac{4b}{a}$$

By AM-GM inequality, for positive numbers,

$$\frac{9a}{b}+\frac{4b}{a}\geq\sqrt[2]{\frac{9a}{b}\cdot\frac{4b}{a}}$$
$$\Rightarrow\quad 2\sqrt{36}=2\cdot6=12$$

So, $13+\frac{9a}{b}+\frac{4b}{a}=13+12=25$

But, since $(a+b)\left(\frac{4}{a}+\frac{9}{b}\right)=(a+b)\cdot1$

$=a+b \quad$ (Using Eq. (i))

$\therefore$ The minimum value of $a+b$ is 25 and this occurs when $\frac{9a}{b}=\frac{4b}{a}$

$$\Rightarrow\quad 9a^2=4b^2 \text{ or } 3a=2b \quad (\because a,b>0)$$
$$\Rightarrow\quad b=\frac{3}{2}a$$

So, $\frac{4}{a}+\frac{9}{b}=1$

$$\Rightarrow\frac{4}{a}+\frac{9}{\frac{3}{2}a}=1\Rightarrow\frac{4}{a}+\frac{18}{3a}=1$$
$$\Rightarrow\frac{4}{a}+\frac{6}{a}=1\Rightarrow\frac{10}{a}=1$$
$$\Rightarrow\quad a=10$$

Then, $b=\frac{3}{2}a=\frac{3}{2}\times10=15$

So, the minimum value of $OA+OB$ is $a+b=10+15=25$

Question46

If $4x^2+12xy+9y^2+2gx+2fy-1=0$ represent a pair of parallel lines, then

Options:

A.

$\frac{f}{g}+\frac{g}{f}+\frac{13}{6}=0$

B.

$f^2+g^2=fg$

C.

$f^2+g^2=6fg$

D.

$\frac{f}{g}+\frac{g}{f}=\frac{13}{6}$

Answer: D

Solution:

Given equation,

$4x^2+12xy+9y^2+2gx+2fy-1=0$

Since, general second-degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, if

$\Delta=abc+2fgh-bg^2-ch^2-af^2=0$

Here, $a=4, b=9, h=6$

So, $\Delta=4\times 9\times c+2fg\times 6-9g^2$

$$\begin{aligned} & -c(6)^2-4f^2=0 \\ & =36c+12fg-9g^2-36c-4f^2=0 \\ & =12fg-9g^2-4f^2=0 \\ \Rightarrow \quad & -(3g-2f)^2=0 \Rightarrow 3g-2f=0 \\ \Rightarrow \frac{g}{f} & =\frac{2}{3} \text{ and } \frac{f}{g}=\frac{3}{2} \end{aligned}$$

So, $\frac{f}{g}+\frac{g}{f}=\frac{3}{2}+\frac{2}{3}=\frac{9+4}{6}=\frac{13}{6}$

Question47

If the equation of the circle passing through the points $(-1,0),(-1,1),(1,1)$ is $ax^2+ay^2+2gx+2fy-2=0$, then $a=$

Options:

A.

1

B.

-1

C.

2

D.

-2

Answer: C

Solution:

Given equation is

$ax^2 + ay^2 + 2gx + 2fy - 2 = 0$

Put $(-1, 0)$ into this equation, we get

$$a(-1)^2 + a(0)^2 + 2g(-1) + 2f(0) - 2 = 0$$
$$\Rightarrow \quad a - 2g - 2 = 0 \quad \ldots(i)$$

Put $(-1, 1)$, we get

$$a(-1)^2 + a(1)^2 + 2g(-1) + 2f(1) - 2 = 0$$
$$\Rightarrow \quad a + a - 2g + 2f - 2 = 0$$

$$\Rightarrow \quad 2a - 2g + 2f - 2 = 0$$
$$\Rightarrow \quad a - g + f - 1 = 0 \quad \ldots(ii)$$

Put (1, 1), we get

$$a(1)^2 + a(1)^2 + 2g(1) + 2f(1) - 2 = 0$$
$$\Rightarrow \quad a + g + f - 1 = 0 \quad \ldots(iii)$$

Subtract Eq. (ii) from Eq. (iii),

$$(a + g + f - 1) - (a - g + f - 1) = 0$$
$$\Rightarrow \quad 2g = 0$$
$$\Rightarrow \quad g = 0$$

Put $g = 0$ into Eq. (i), we get

$$a - 2(0) - 2 = 0$$
$$\Rightarrow \quad a = 2$$

Question48

For the circle $x-2=5\cos\theta, y+1=5\sin\theta$, where θ is the perimeter, the line $x=1+\frac{r}{2}, y=-2+\frac{\sqrt{3}}{2}r$ where r is the perimeter, is a

Options:

A.

Chord of the circle other than diameter

B.

Tangent of the circle

C.

Diameter of the circle

D.

Line that does not meet the circle

Answer: A

Solution:

Given circle, $x-2=5\cos\theta, y+1=5\sin\theta$, where θ is the perimeter.

$$\Rightarrow (x-2)^2+(y+1)^2=(5\cos\theta)^2+(5\sin\theta)^2$$
$$=25\left(\cos^2\theta+\sin^2\theta\right)=25$$

$\therefore$ The circle has a centre at $(h,k)=(2,-1)$ and a radius, $r_c=5$

Now, the line equation is $x=1+\frac{r}{2}$ and $y=-2+\frac{\sqrt{3}}{2}r$. This is parametric form.

Let $x_1=1, y_1=-2$

The direction vector of the line is $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

So, the slope of the line, $m=\frac{\Delta y}{\Delta x}$

$$\Rightarrow \quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$

Now, the equation of line is

$$\begin{aligned} & y-y_1=m\,(x-x_1) \\ \Rightarrow\quad & y-(-2)=\sqrt{3}(x-1) \\ \Rightarrow\quad & y+2=\sqrt{3}x-\sqrt{3} \\ \Rightarrow\quad & \sqrt{3}x-y-(2+\sqrt{3})=0 \end{aligned}$$

Now, calculate the distance from the center of the circle to line,

$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

Here, $(x_0, y_0)=(2,-1)$ and the line is $\sqrt{3}x-y-(2+\sqrt{3})=0$

So, $A=\sqrt{3}, B=-1, C=-(2+\sqrt{3})$

$$\begin{aligned} d &= \frac{|\sqrt{3}(2)+(-1)(-1)-(2+\sqrt{3})|}{\sqrt{(\sqrt{3})^2+(-1)^2}} \\ &= \frac{|2\sqrt{3}+1-2-\sqrt{3}|}{\sqrt{3+1}} \\ &= \frac{\sqrt{3}-1}{2} \approx 0.366 < 5 \end{aligned}$$

So, the line intersects the circle at two distinct points, meaning it is a secant to the circle.

So, the line is a chord of circle other than diameter.

Question49

If $x-2y=0$ is a tangent drawn at a point P on the circle $x^2+y^2-6x+2y+c=0$, then the distance of the point $(6,3)$ from P is

Options:

A.

$\sqrt{5}$

B.

$2\sqrt{5}$

C.

$4\sqrt{5}$

D.

$5\sqrt{2}$

Answer: B

Solution:

The equation of the circle is

$$
\begin{aligned}
& x^2+y^2-6x+2y+c=0 \\
\Rightarrow \quad & (x-3)^2+(y+1)^2=10-c
\end{aligned}
$$

So, the center of the circle is $C(3,-1)$ and the radius is $r^2=10-C$

Now, since the equation of tangent is

$x-2y=0$

Slop, $m_t=1/2$

Slope of the radius, connecting the centre $(3,-1)$ to the point of tangency

$P\left(x_p, y_p\right)$ is $m_r=\frac{y_p-(-1)}{x_p-3}=\frac{y_p+1}{x_p-3}$

Now, since radius is perpendicular to the tangent, so

$$
\begin{aligned}
& m_t \cdot m_r=-1 \\
\Rightarrow \quad & \frac{1}{2} \cdot \frac{y_p+1}{x_p-3}=-1 \Rightarrow y_p+1=-2x_p+6 \\
\Rightarrow \quad & y_p=-2x_p+5
\end{aligned}
$$

But, P lies on the tangent line, its coordinates satisfy $x_p-2y_p=0$

$$
\begin{aligned}
\Rightarrow \quad & x_p=2y_p \\
\text{So,} \quad & y_p=-2x_p+5 \\
\Rightarrow \quad & -2\left(2y_p\right)+5=-4y_p+5 \\
\Rightarrow \quad & 5y_p=5 \Rightarrow y_p=1
\end{aligned}
$$

So, $x_p=2y_p=2\times 1=2$

So, the point P is $(2,1)$.

Now, the distance between $(6,3)$ and $(2,1)$ is

$$
\begin{aligned}
& d=\sqrt{(2-6)^2+(1-3)^2} \\
& \Rightarrow \sqrt{(-4)^2+(-2)^2} \Rightarrow \sqrt{16+4}=\sqrt{20}=2\sqrt{5}
\end{aligned}
$$

$\therefore$ The distance of the point $(6,3)$ from P is $2\sqrt{5}$.

Question50

If A, B are the points of contact of the tangents drawn from the point $(-3, 1)$ to the circle $x^2 + y^2 - 4x + 2y - 4 = 0$, then the equation of the circumcircle of the $\triangle PAB$ is

Options:

A.

$x^2 + y^2 - 6x + 2y - 6 = 0$

B.

$x^2 + y^2 - x + 7 = 0$

C.

$x^2 + y^2 + x - 7 = 0$

D.

$x^2 + y^2 + 6x - 2y - 6 = 0$

Answer: C

Solution:

The equation of the circle is

$x^2 + y^2 - 4x + 2y - 4 = 0$

Comparing this with the general form

$x^2 + y^2 + 2gx + 2fy + c = 0$

We have

$$\begin{aligned} & 2g = -4 \\ \Rightarrow \quad & g = -2 \text{ and } 2f = 2 \\ \Rightarrow \quad & f = 1 \end{aligned}$$

So, the center of the circle C is

$(-g, -f) = (2, -1)$

And, the radius is $r = \sqrt{g^2 + f^2 - c}$

$$\begin{aligned} & = \sqrt{(-2)^2 + (1)^2 - (-4)} \\ & = \sqrt{4 + 1 + 4} = \sqrt{9} = 3 \end{aligned}$$

The point from which tangents are drawn is $P = (-3, 1)$

Now, the circumcircle of $\triangle PAB$ is the circle with PC as its diameter. The center of this circumcircle is the mid-point of PC.

Mid-point $M = \left(\frac{-3+2}{2}, \frac{1+(-1)}{2}\right)$

$\Rightarrow \left(\frac{-1}{2}, \frac{0}{2}\right) = \left(\frac{-1}{2}, 0\right)$

Distance $PC = \sqrt{(2-(-3))^2 + (-1-1)^2}$

$$\begin{aligned} \Rightarrow \quad & \sqrt{5^2 + (-2)^2} \\ \Rightarrow \quad & \sqrt{25+4} = \sqrt{29} \end{aligned}$$

$\therefore$ The radius of the circumcircle

$= \frac{1}{2} \cdot$ distance $PC = \frac{\sqrt{29}}{2}$

Now, the equation of a circle with center (h, k) and radius R is

$(x-h)^2 + (y-k)^2 = R^2$

Here, $(h, k) = \left(\frac{-1}{2}, 0\right)$ and $R = \frac{\sqrt{29}}{2}$

So, $\left(x - \left(-\frac{1}{2}\right)\right)^2 + (y-0)^2 = \left(\frac{\sqrt{29}}{2}\right)^2$

$$\begin{aligned} \Rightarrow \quad & \left(x + \frac{1}{2}\right)^2 + y^2 = \frac{29}{4} \\ \Rightarrow \quad & x^2 + x + \frac{1}{4} + y^2 = \frac{29}{4} \\ \Rightarrow \quad & x^2 + y^2 + x - \frac{28}{4} = 0 \\ \Rightarrow \quad & x^2 + y^2 + x - 7 = 0 \end{aligned}$$

Question51

If the angle between the circles $x^2 + y^2 - 2x + ky + 1 = 0$ and $x^2 + y^2 - kx - 2y + 1 = 0$ is $\cos^{-1}\left(\frac{1}{4}\right)$ and $k < 0$, then the point which lies on the radical axis of the given circle is

Options:

A.

$(1, -3)$

B.

$(-1, 3)$

C.

$(-1,-3)$

D.

$(1,3)$

Answer: A

Solution:

For $S_1: x^2+y^2-2x+ky+1=0$,

Center $C_1=\left(1,-\frac{k}{2}\right)$

Radius, $r_1=\sqrt{1^2+\left(-\frac{k}{2}\right)^2-1}=\sqrt{\frac{k^2}{4}}=\frac{k}{2}$

For $S_2: x^2+y^2-kx-2y+1=0$

Center $C_2=\left(\frac{k}{2},1\right)$

Radius, $r_2=\sqrt{\left(\frac{k}{2}\right)^2+1^2-1}$

$\sqrt{\frac{k^2}{4}}=\frac{k}{2}$

Since, angle θ between two circles is given by $\cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}$, where d is the distance between the centers.

Given, $\theta=\cos^{-1}\left(\frac{1}{4}\right)$

$$\begin{aligned}\Rightarrow \quad \cos\theta &= \frac{1}{4}\\ d^2 &= \left(1-\frac{k}{2}\right)^2+\left(\frac{-k}{2}-1\right)^2\\ &= 1-k+\frac{k^2}{4}+1+k+\frac{k^2}{4}\\ &= 2+\frac{k^2}{2}\end{aligned}$$

Since $k<0, r_1=\frac{-k}{2}$ and $r_2=\frac{-k}{2}$

Substituting into the formula

$\frac{1}{4}=\frac{\left(2+\frac{k^2}{2}\right)-\left(\frac{-k}{2}\right)^2-\left(\frac{-k}{2}\right)^2}{2\left(\frac{-k}{2}\right)\left(\frac{-k}{2}\right)}$

$\Rightarrow \quad \frac{1}{4}=\frac{2+\frac{k^2}{2}-\frac{k^2}{4}-\frac{k^2}{4}}{2\left(\frac{k^2}{4}\right)}$

$\Rightarrow \quad \frac{1}{4}=\frac{4}{k^2}$

$\Rightarrow \quad k^2=16$

$\Rightarrow \quad k= \pm 4$

But since $k<0, k=-4$

Now, the equation of the radical axis is

$S_1-S_2=0$
$\left(x^2+y^2-2x+ky+1\right)-\left(x^2+y^2-kx\right.$
$\left.-2y+1\right)=0$
$\Rightarrow -2x+ky+kx+2y=0$
$\Rightarrow -6x-2y=0 \quad (\because k=-4)$
$\Rightarrow 3x+y=0$

The points satisfying this equation are $(-1,3)$ and $(1,-3)$ lies on the radical axis.

Question52

A circle C passing through the point $(1,1)$ bisects the circumference of the circle $x^2+y^2-2x=0$. If C is orthogonal to the circle $x^2+y^2+2y-3=0$, then the centre of the circle C is

Options:

A.

$\left(-\frac{1}{2}, 0\right)$

B.

$\left(\frac{5}{2}, 0\right)$

C.

$\left(0, \frac{5}{2}\right)$

D.

$\left(0,-\frac{1}{2}\right)$

Answer: B

Solution:

Let the equation of circle C be

$x^2+y^2+2gx+2fy+c=0$

Since, it passes through $(1,1)$ then

$$1^2+1^2+2g+2f+c=0$$
$$\Rightarrow \quad 2+2g+2f+c=0 \quad \ldots(i)$$

Now, $S_1 : x^2+y^2+2gx+2fy+c=0$

and $S_2 = x^2+y^2-2x=0$

So, $S_1 - S_2 = 0$ is the common chord.

$\Rightarrow (2g+2x+2fy+c=0$

Now, center of $s_2 = x^2+y^2-2x=0$ is $(1,0)$

Since, the common chord is a diameter of S_2, then centre of S_2 must lies on the common chord.

$$\therefore \quad (2g+2(l)+2f(0)+c=0$$
$$\Rightarrow \quad 2g+2+c=0 \quad \ldots(ii)$$

Also, circle C is orthogonal to

$x^2+y^2+2y-3=0$

Here, $g_1=g, f_1=f, c_1=c$ and $g_2=0$,

$f_2=1, c_2=-3 \quad \ldots(iii)$

Using the orthogonality condition

$$2g_1g_2+2f_1f_2=c_1+c_2$$
$$\Rightarrow \quad 2g(0)+2f(1)=c+(-3)$$
$$\Rightarrow \quad 2f=c-3$$

From Eq. (ii), $c=-2g-2$

Put this in Eq. (iii), we get

$$2f=c-3=-2g-2-3$$
$$\Rightarrow \quad 2f=-2g-5 \quad \ldots(iv)$$

Substitute $c=-2g-2$ into Eq. (i), we get

$$\Rightarrow \quad 2+2g+2f-2g-2=0$$
$$\Rightarrow \quad 2f=0$$
$$\Rightarrow \quad f=0$$

Put $f=0$ in Eq. (iv), we get

$$2(0)=-2g-5$$
$$\Rightarrow \quad 2g=-5$$
$$\Rightarrow \quad g=\frac{-5}{2}$$

The center of circle C is

$$(-g,-f)=\left(-\left(-\frac{5}{2}\right),0\right)$$

$$\Rightarrow\left(\frac{5}{2},0\right)$$

Question53

If the normal drawn at $P(8,16)$ to the parabola $y^2=32x$ meets the parabola again at Q, then the equation of the tangent drawn at Q to the parabola is

Options:

A.

$x+3y+72=0$

B.

$x-y-120=0$

C.

$3x-y-264=0$

D.

$x+y-24=0$

Answer: A

Solution:

Given parabola is $y^2=32x$,

So, $4a=32$

$\Rightarrow \quad a=8$

The point P is $(8,16)$. This in parametric form as $\left(at^2,2at\right)$

So, $8t^2=8$

$\Rightarrow \quad t^2=1\Rightarrow t=\pm1$

And, $2at=2(8)t=16t=16$

$\Rightarrow \quad t=1$

So, the perimeter for P is $t_1 = 1$.

The equation of the normal to the parabola at a point $P\left(at_1^2, 2at_1\right)$ is

$$y + t_1 x = 2at_1 + at_1^3$$
$$\Rightarrow \quad y + (1)x = 2(8)(1) + 8(1)^3$$
$$\Rightarrow \quad y + x = 16 + 8 = 24$$
$$\Rightarrow \quad y = 24 - x$$

Substitute this into parabola equation

$$y^2 = 32x$$
$$\Rightarrow \quad (24 - x)^2 = 32x$$
$$\Rightarrow \quad 576 - 48x + x^2 = 32x$$
$$\Rightarrow \quad x^2 - 80x + 576 = 0$$

Since, $x = 8$ is one root corresponding to point P. Let the other root be x_Q.

Using the sum of roots, $8 + x_Q = 80$

$$\Rightarrow x_Q = 72$$
$$\text{So, } y = 24 - x_Q$$
$$= 24 - 72 = -48$$

So, the coordinates of Q are $(72, -48)$

Now, the equation of the tangent to parabola $y^2 = 4ax$ at a point (x_1, y_1) is

$$yy_1 = 2a\left(x + x_1\right)$$
$$\Rightarrow y(-48) = 2(8)(x + 72) = 16(x + 72$$
$$\Rightarrow -3y = x + 72 \Rightarrow x + 3y + 72 = 0$$

So, the equation of the tangent drawn at Q to the parabola is $x + 3y + 72 = 0$

Question54

The focal distance of a point $(5, 5)$ on the parabola $x^2 - 2x - 4y + 5 = 0$ is

Options:

A.

5

B.

8

C.

10

D.

12

Answer: A

Solution:

Given equation is

$$\begin{aligned} & x^2-2x-4y+5=0 \\ \Rightarrow \quad & (x-1)^2-4y+4=0 \\ \Rightarrow \quad & (x-1)^2=4(y-1) \end{aligned}$$

So, $(h, k)=(1,1)$ are the vertex

And $4a=4$

$\Rightarrow a=1$

Since, the parabola open upwards (because the y term is positive and x term is squared), the focus is $(h, k+a)=(1,1+1)=(1,2)$ and the directrix is the line $y=k-a=1-1=0$, which is the X-axis.

Since, focal distance of a point on a parabola is its distance from the directrix.

The point $(5,5)$ and directrix is $y=0$.

The distance from a point (x_0, y_0) to a horizontal line $y=c$ is $|y_0-c|$.

$\therefore$ Focal distance $=|5-0|=5$

So, the focal distance of the point $(5,5)$ on the parabola $x^2-2x-4y+5=0$ is 5 .

Question55

If S and S' are the foci of an ellipse $\frac{x^2}{169}+\frac{y^2}{144}=1$ and the point B lying on positive Y-axis is one end of its minor axis, then the incentre of the $\triangle SBS'$ is

Options:

A.

$\left(0, \frac{10}{3}\right)$

B.

$\left(\frac{13}{3}, \frac{10}{3}\right)$

C.

$\left(\frac{10}{3}, \frac{13}{3}\right)$

D.

$\left(0, \frac{13}{3}\right)$

Answer: A

Solution:

The equation of ellipse is

$\frac{x^2}{169} + \frac{y^2}{144} = 1$

So, $a^2 = 169, b^2 = 144$

$$\begin{aligned} \Rightarrow \quad & a = 13, b = 12 \\ \therefore \quad & c^2 = a^2 - b^2 \Rightarrow 169 - 144 = 25 \\ \Rightarrow \quad & c = 5 \end{aligned}$$

So, the foci are at $(\pm c, 0)$. So, $S = (5, 0)$ and $S' = (-5, 0)$

Since, the point B is one end of the minor axis lying on the positive Y-axis.

So, $B = (0, \pm b) = (0, 12)$

$\therefore$ The vertices of the triangle are $S(5, 10), S'(-5, 0)$ and $B(0, 12)$

$$\begin{aligned} SS' &= \sqrt{(-5-5)^2 + (0-0)^2} \\ &= \sqrt{(-10)^2} = 10 \\ SB &= \sqrt{(0-5)^2 + (12-0)^2} \\ &= \sqrt{25+144} = \sqrt{169} = 13 \\ S'B &= \sqrt{(0-(-5))^2 + (12-0)^2} \\ &= \sqrt{25+144} = \sqrt{169} = 13 \end{aligned}$$

Since, $SB = S'B$, So SBS' is an isosceles triangle.

Now, the incenter (I_x, I_y) of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and opposite side lengths a, b and c is

$I_x = \frac{ax_1+bx_2+cx_3}{a+b+c}, I_y = \frac{ay_1+by_2+cy_3}{a+b+c}$

Here, $(x_1, y_1) = S(5, 0), (x_2, y_2) = S'(-5, 0), (x_3, y_3) = B(0, 12)$ and $a = S'B = 13$, $b = SB = 13, C = SS' = 10$

$\therefore \quad I_x = \frac{13(5)+13(-5)+10(0)}{13+13+10}$

$\Rightarrow \quad \frac{65-65+0}{36} = 0$

$I_y = \frac{13(0)+13(0)+10(12)}{13+13+10}$

$\Rightarrow \quad \frac{0+0+120}{36} = \frac{10}{3}$

So, the incenter of the $\Delta SBS'$ is $\left(0, \frac{10}{3}\right)$.

Question56

One of the foci of an ellipse is $(2, -3)$ and its corresponding directrix is $2x + y = 5$. If the eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$, then the coordinates of the other focus are

Options:

A.

$(18, 5)$

B.

$(4, -2)$

C.

$(-2, -5)$

D.

$(-4, -6)$

Answer: C

Solution:

Let the given focus be $F_1(2, -3)$ and the corresponding directrix be $L_1 : 2x + y - 5 = 0$ and its eccentricity,

$e = \frac{\sqrt{5}}{3}$

Let the other focus be $F_2\left(x_2, y_2\right)$

Now, the distance from $F_1(2, -3)$ to $2x + y - 5 = 0$ is

$d_1 = \frac{|2(2)+(-3)-5|}{\sqrt{2^2+1^2}}$

$= \frac{|4-3-5|}{\sqrt{4+1}} = \frac{|-4|}{\sqrt{5}} = \frac{4}{5}$

Let $P(x, y)$ be any point on the ellipse.

So, $PF_1 = \sqrt{(x-2)^2 + (y+3)^2}$

And the distance from P to the directrix L_1 is

$$PM_1 = \frac{|2x+y-5|}{\sqrt{2^2+1^2}}$$
$$\Rightarrow \quad \frac{|2x+y-5|}{\sqrt{5}}$$

Since, $PF_1 = e \cdot PM_1$

$$\Rightarrow \sqrt{(x-2)^2 + (y+3)^2}$$
$$= \frac{\sqrt{5}}{3} \cdot \frac{|2x+y-5|}{\sqrt{5}}$$
$$\Rightarrow \frac{|2x+y-5|}{3}$$

Squaring on both sides, we get

$$(x-2)^2 + (y+3)^2 = \frac{(2x+y-5)^2}{9}$$
$$\Rightarrow 9\left((x-2)^2 + (y+3)^2\right)$$
$$= 4x^2 + y^2 + 25 + 4xy - 20x - 10y$$
$$= 9x^2 - 36x + 36 + 9y^2 + 54y + 81$$
$$= 4x^2 + y^2 + 25 + 4xy - 20x - 10y$$
$$\Rightarrow 5x^2 + 8y^2 - 4xy - 16x + 64y + 92 = 0$$

This is equation of ellipse.

Now, slope of directrix is $m_D = -2$ and the line joining the two foci is perpendicular to the directrix.

So, $m_F = \frac{-1}{m_D} = \frac{-1}{-2} = \frac{1}{2}$

Let, the other focus be $F_2\left(x_2, y_2\right)$

$$\text{So, } \frac{y_2-(-3)}{x_2-2} = \frac{y_2+3}{x_2-2} = \frac{1}{2}$$
$$\Rightarrow 2\left(y_2+3\right) = x_2 - 2$$
$$\Rightarrow \quad x_2 = 2y_2 + 8$$

We know that $c = ae$ and the distance between the focus and directrix is

$$\frac{a}{e} - c = \frac{a}{e} - ae = \frac{a\left(1-e^2\right)}{e}$$

$$\Rightarrow \quad \frac{a\left(1-\left(\frac{\sqrt{5}}{3}\right)^2\right)}{\frac{\sqrt{5}}{3}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \quad \frac{a\left(1-\frac{5}{9}\right)}{\frac{\sqrt{5}}{3}} = \frac{4}{\sqrt{5}} \Rightarrow a\left(\frac{4}{9}\right) \times \frac{3}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \quad \frac{4a}{3\sqrt{5}} = \frac{4}{\sqrt{5}} \Rightarrow a = 3$$

$$\therefore \quad c = ae = 3 \times \frac{\sqrt{5}}{3} = \sqrt{5}$$

The distance between two foci $= 2c = 2\sqrt{5}$

Let $F_2\left(x_2, y_2\right)$. So, the distance

$$F_1F_2 = \sqrt{\left(x_2-2\right)^2 + \left(y_2+3\right)^2} = 2\sqrt{5}$$

$$\Rightarrow \sqrt{\left(2y_2+8-2\right)^2 + \left(y_2+3\right)^2} = 2\sqrt{5}$$
$$\left(\because x_2 = 2y_2+8\right)$$
$$\Rightarrow \quad \sqrt{4\left(y_2+3\right)^2 + \left(y_2+3\right)^2} = 2\sqrt{5}$$
$$\Rightarrow \quad \sqrt{5\left(y_2+3\right)^2} = 2\sqrt{5}$$
$$\Rightarrow \quad \sqrt{5}\left|y_2+3\right| = 2\sqrt{5} \Rightarrow \left|y_2+3\right| = 2$$

So, $y_2 + 3 = 2$ or $y_2 + 3 = -2$

$$\Rightarrow \quad y_2 = -1 \text{ or } y_2 = -5$$
$$\therefore \quad x_2 = 2(-1) + 8 = 6 \text{ and}$$
$$x_2 = 2(-5) + 8 = -2$$

So, $F_2(6, -1)$ and $F_2(-2, -5)$

Line $2x + y - 5 = 0$

For $F_1(2, -3), 2(2) + (-3) - 5 = 4 - 8 = -4$

For $F_2(6, -1), 2(6) + (-1) - 5 = 12 - 6 = 6$

Since, -4 and 6 have opposite signs, so foci of an ellipse is $F_2(-2, -5)$.

Question57

If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is $\frac{36}{13}$ and its eccentricity is $\frac{\sqrt{13}}{3}$, then $a - b =$

Options:

A.

4

B.

3

C.

2

D.

1

Answer: D

Solution:

Since, the product of perpendicular distance from any point on the hyperbola to its asymptotes $= \frac{36}{13}$

So, $\frac{a^2b^2}{a^2+b^2} = \frac{36}{13}$ $\quad\quad\quad\quad\ldots(i)$

Also, $e = \sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow \frac{\sqrt{13}}{3} = \sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow \frac{13}{9} = 1+\frac{b^2}{a^2}$

(Squaring on both sides)

$$\Rightarrow \frac{b^2}{a^2} = \frac{13}{9} - 1 = \frac{4}{9}$$
$$\Rightarrow b^2 = \frac{4}{9}a^2$$

$\Rightarrow b = \frac{2}{3}a \quad (\because a, b > 0)$

Substitute this into Eq. (i), we get

$$\frac{a^2 \cdot \left(\frac{2}{3}a\right)^2}{a^2 + \frac{4}{9}a^2} = \frac{36}{13}$$
$$\Rightarrow \frac{\frac{4}{9}a^4}{\frac{13}{9}a^2} = \frac{36}{13} \Rightarrow \frac{4}{13}a^2 = \frac{36}{13}$$
$$\Rightarrow 4a^2 = 36$$
$$\Rightarrow a^2 = 9$$

$\Rightarrow a = 3 \quad (\because a > 0)$

And $b=\frac{2}{3}a=\frac{2}{3}\times 3=2$

Now, $a-b=3-2=1$

Question58

If $A(0,3,4), B(1,5,6), C(-2,0,-2)$ are the vertices of a $\triangle ABC$ and the bisector of angle A meets the side BC at D, then $AD=$

Options:

A.

$\frac{\sqrt{21}}{5}$

B.

$\frac{\sqrt{42}}{10}$

C.

10

D.

4

Answer: B

Solution:

$$\begin{aligned} AB &= \sqrt{(1-0)^2+(5-3)^2+(6-4)^2} \\ &= \sqrt{1+4+4}=\sqrt{9}=3 \\ AC &= \sqrt{(-2-0)^2+(0-3)^2+(-2-4)^2} \\ &= \sqrt{4+9+36}=\sqrt{49}=7 \end{aligned}$$

Using the angle bisector theorem,

$\frac{BD}{DC}=\frac{AB}{AC}=\frac{3}{7}$

Since, D divides BC in the ratio $3:7$, using the section formula

$$D=\left(\frac{7\cdot 1+3\cdot(-2)}{7+3},\frac{7\cdot 5+3\cdot 0}{7+3},\frac{7\cdot 6+3\cdot(-4)}{7+3}\right)$$
$$\Rightarrow\left(\frac{7-6}{10},\frac{35}{10},\frac{42-6}{10}\right)$$
$$\Rightarrow\left(\frac{1}{10},\frac{35}{10},\frac{36}{10}\right)=(0.1,35,3.6)$$

Now,

$$AD=\sqrt{(0.1-0)^2+(3.5-3)^2+(3.6-4)^2}$$
$$\Rightarrow\sqrt{(0.1)^2+(0.5)^2+(-0.4)^2}$$
$$\Rightarrow\sqrt{0.01+0.25+0.16}=\sqrt{0.42}$$
$$\Rightarrow\sqrt{\frac{42}{100}}=\frac{\sqrt{42}}{10}$$

Question59

If the direction cosines of two lines satisfy the equation $2l+m-n=0, l^2-2m^2+n^2=0$ and θ is the angle between the lines, then $\cos\theta=$

Options:

A.

$\frac{1}{5}$

B.

$\frac{\pi}{4}$

C.

$\frac{2}{3}$

D.

$\frac{\pi}{3}$

Answer: A

Solution:

Given two lines $2l+m-n=0$

$\Rightarrow \quad n=2l+m$ and $l^2-2m^2+n^2=0$

$\Rightarrow \quad l^2-2m^2+(2l+m)^2=0$

$\Rightarrow \quad l^2-2m^2+4l^2+4lm+m^2=0$

$\Rightarrow \quad 5l^2+4lm-m^2=0$

$\Rightarrow \quad 5\left(\frac{l}{m}\right)^2+4\left(\frac{l}{m}\right)-1=$

Let $x=\frac{l}{m}, 5x^2+4x-1=0$

$\Rightarrow \quad x=\frac{-4 \pm \sqrt{4^2-4(9)(-1)}}{2(9)}$

$\Rightarrow \quad \frac{-4 \pm \sqrt{36}}{10}=\frac{-4 \pm 6}{10}$

$\therefore \quad x_1=\frac{2}{10}$ and $x_2=\frac{-10}{10}=-1$

Case I $\frac{l}{m}=\frac{1}{5}$

$\Rightarrow \quad m=5l$

So, $n=2l+m=2l+5l=7l$

Direction cosine (l_1, m_1, n_1) are proportional to $(l, 5l, 7l)$, i.e., $(1,5,7)$

Case II $\frac{l}{m}=-1$

$\Rightarrow \quad m=-l$

So, $n=2l+m=2l-l=l$

Direction cosine (l_2, m_2, n_2) are proportional to $(l,-l,l)$ i.e., $(1,-1,1)$

Now,

$$\cos \theta=\frac{l_1 l_2+m_1 m_2+n_1 n_2}{\sqrt{l_1^2+m_1^2+n_1^2} \cdot \sqrt{l_2^2+m_2^2+n_2^2}}$$

$$\Rightarrow \frac{1 \times 1+5 \times(-1)+7 \times 1}{\sqrt{1^2+5^2+7^2} \cdot \sqrt{1^2+(-1)^2+1^2}}$$

$$\Rightarrow \frac{1-5+7}{\sqrt{1+25+49} \cdot \sqrt{1+1+!}}$$

$$\Rightarrow \frac{3}{\sqrt{75} \cdot \sqrt{3}}=\frac{3}{5 \cdot 3} \Rightarrow \frac{3}{15}=\frac{1}{5}$$

$\therefore \cos \theta=\frac{1}{5}$

Question60

If the equation of the plane passing through the points $(2, 1, 2), (1, 2, 1)$ and perpendicular to the plane $2x - y + 2z = 1$ is $ax + by + cz + d = 0$, then $\frac{a+b}{c+d} =$

Options:

A.

0

B.

1

C.

-1

D.

2

Answer: C

Solution:

Let $P_1 = (2, 1, 2)$ and $P_2 = (1, 2, 1)$

So, $\mathbf{P}_1\mathbf{P}_2 = P_2 - P_1$

$$\Rightarrow \quad (1, 2, 1) - (2, 1, 2)$$
$$\Rightarrow \quad (1 - 2, 2 - 1, 1 - 2$$
$$\Rightarrow \quad (-1, 1, -1)$$

The plane $2x - y + 2z = 1$ has a normal vector $\mathbf{n}_1 = (2, -1, 2)$.

Since, the required plane is perpendicular to $2x - y + 2z = 1$, its normal vector $\mathbf{n} = (a, b, c)$ must be perpendicular to $\mathbf{n}_1$.

So, $\mathbf{n}$ must be perpendicular to $\mathbf{P}_1\mathbf{P}_2$.

$\therefore$ $\mathbf{n}$ is parallel to the cross product of $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{n}_1$.

$$\mathbf{n} = \mathbf{P}_1\mathbf{P}_2 \times \mathbf{n}_1 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= (2 - 1)\hat{\mathbf{i}} - (-2 + 2\hat{\mathbf{j}} + (1 - 2\hat{\mathbf{k}}$$
$$= \hat{\mathbf{i}} - \hat{\mathbf{k}} = \langle 1, 0, -1 \rangle$$

Now, equation of the point $(2, 1, 2)$ and the normal vector $(1, 0, -1)$ is

$1(x-2+0(y-1)-1(z-2)=0$
$\Rightarrow \quad x-2-z+2=0$
$\Rightarrow \quad x-z=0$

Comparing this with $ax+by+cz+d=0$, we have

$$a=1, b=0, c=-1, d=0$$
$$\therefore \frac{a+b}{c+d}=\frac{1+0}{-1+0}=-1$$

Question61

If $[x]$ is the greatest integer function, then

$$\lim_{x \rightarrow 3} \frac{(3-|x|+\sin|3-x|) \cos[9-3x]}{|3-x|[3x-9]}$$

Options:

A.

0

B.

1

C.

2

D.

-2

Answer: D

Solution:

Since, $x \rightarrow 3^{-}$, we have $x<3$.

$\therefore |x|=x$ and $|3-x|=3-x$

As, $x \rightarrow 3^{-}, 3x \rightarrow 9^{-}$, so $9-3x \rightarrow 0^{+}$

Thus, $[9-3x]=0$ for x slightly less than 3 and $[3x-9]=-1$ for x slightly less than 3 .

$$\therefore \lim_{x \to 3^-} \frac{(3 - x + \sin(3 - x))\cos(0)}{(3 - x)(-1)}$$

$$= \lim_{x \to 3^-} \frac{(3 - x + \sin(3 - x))}{(3 - x)(-1)}$$

Let $t = 3 - x$.

As, $x \to 3^-, t \to 0^+$

$$\text{So,} \quad \lim_{x \to 3^-} \frac{(3 - x + \sin(3 - x))}{(3 - x)(-1)}$$

$$\Rightarrow \quad \lim_{t \to 0^+} \frac{t + \sin t}{-t}$$

$$\Rightarrow \quad \lim_{t \to 0^+} \left(-1 - \frac{\sin t}{t}\right)$$

$$\Rightarrow \quad -1 - 1 \quad \left[\because \lim_{t \to 0} \frac{\sin t}{t} = 1\right]$$

$$\Rightarrow \quad -2$$

$$\therefore \lim_{x \to 3^-} \frac{(3 - |x| + \sin|3 - x|)\cos[9 - 3x]}{|3 - x|[3x - 9]}$$

$$= -2$$

Question62

Let ' a ' be a positive real number. If a real valued function

$$f(x) = \begin{cases} \frac{6^x - 3^x - 2^x + 1}{1 - \cos\left(\frac{x}{a}\right)} & \text{if } x \neq 0 \\ \log 3 \log 4 & \text{if } x = 0 \end{cases}$$ **is continuous at $x = 0$, then $a =$**

Options:

A.

1

B.

2

C.

3

D.

4

Answer: A

Solution:

Given,

$$f(x)=\begin{cases}\frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)}, & x\neq 0\\ \log 3\log 4, & x=0\end{cases}$$

Now, $f(0)=\log 3\log 4$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)}$

$$\Rightarrow \lim_{x\to 0}\frac{(3^x-1)(2^x-1)}{1-\cos\left(\frac{x}{a}\right)}$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{\frac{3^x-1}{x}\cdot\frac{2^x-1}{x}}{\frac{1-\cos\left(\frac{x}{a}\right)}{x^2}}\right]$$

$$\Rightarrow \frac{(\ln 3)(\ln 2)}{\frac{1}{(2a^2)}}\left[\begin{array}{l}\because \lim_{x\to 0}\frac{a^x-1}{x}=\ln a\\ \text{and } \lim_{x\to 0}\frac{1-\cos kx}{x^2}=\frac{k^2}{2}\end{array}\right]$$

$$=2a^2(\ln 3)\ln(2)$$

Since, given function is continuous

So, $\lim_{x\to 0} f(x)=f(0)$

$$\begin{aligned}&\Rightarrow \quad 2a^2(\ln 3)(\ln 2)=(\ln 3)(\ln 4)\\&\Rightarrow \quad (\ln 3)(\ln 2)^2=2(\ln 3)(\ln 2)\\&\Rightarrow \quad 2a^2=2\Rightarrow a^2=1\end{aligned}$$

$\Rightarrow \quad a=1 \quad (\because a$ is a positive real number $)$

Question63

If $f(x)=\sqrt{\cos^{-1}\sqrt{1-x^2}}$, then $f'\left(\frac{1}{2}\right)=$

Options:

A.

$\sqrt{\frac{2}{\pi}}$

B.

$\sqrt{\frac{\pi}{2}}$

C.

$-\sqrt{\frac{2}{\pi}}$

D.

$-\sqrt{\frac{\pi}{2}}$

Answer: A

Solution:

Given, $f(x)=\sqrt{\cos^{-1}\sqrt{1-x^2}}$

Let $x=\sin\theta$.

Then, $\sqrt{1-x^2}=\sqrt{1-\sin^2\theta}$

$\Rightarrow \quad \sqrt{\cos^2\theta}=|\cos\theta|$

Since, $\theta\in\left[0,\frac{\pi}{2}\right]$, so $\cos\theta\geq 0$

$\therefore \quad \sqrt{1-x^2}=\cos\theta$

So, $f(x)=\sqrt{\cos^{-1}(\cos\theta)}$

$$\Rightarrow \quad \sqrt{\theta}=\sqrt{\sin^{-1}(x)}$$

$$f'(x)=\frac{d}{dx}\sqrt{\sin^{-1}(x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}\cdot\frac{1}{2\sqrt{\sin^{-1}(x)}}$$

$$f'(x)=\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\cdot\frac{1}{2\cdot\sqrt{\sin^{-1}\frac{1}{2}}}$$

$$=\frac{1}{\frac{\sqrt{3}}{2}}\cdot\frac{1}{2\cdot\sqrt{\frac{\pi}{6}}}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{6}}{2\sqrt{\pi}}\Rightarrow\frac{\sqrt{2}}{\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}$$

$$\therefore \quad f'(x)=\sqrt{\frac{2}{\pi}}$$

Question64

If $y=f(\cosh x)$ and $f'(x)=\log\left(x+\sqrt{x^2-1}\right)$, then $\frac{d^2y}{dx^2}=$

Options:

A.

$\sinh x + x \cosh x$

B.

$x \sinh x$

C.

$\log\left(x + \sqrt{x^2 + 1}\right)$

D.

$\frac{x\left(2\sqrt{x^2-1}+1\right)}{\sqrt{x^2-1}\left(x^2+\sqrt{x^2-1}\right)}$

Answer: A

Solution:

Given, $y = f(\cos hx)$ and

$f'(x) = \log\left(x + \sqrt{x^2 - 1}\right)$

Let $u = \cos hx$, so $y = f(u)$

$$\begin{aligned} \therefore \quad \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= f'(u) \cdot \frac{d}{dx}(\cos hx) \\ &= f'(\cos hx) \cdot \sin hx \end{aligned}$$

Since, $f'(x) = \log\left(x + \sqrt{x^2 - 1}\right)$

$$\begin{aligned} \therefore f'(\cosh x) &= \log\left(\cosh x + \sqrt{\cosh h^2 x - 1}\right) \\ &= \log(\cosh x + |\sinh x|) \end{aligned}$$

Since, $x > 0, \sin h > 0$

So, $f'(\cos hx) = \log(\cos hx + \sin hx)$

$\Rightarrow \log e^x = x \quad [\because \cosh x + \sinh x = e^x]$

So, $\frac{dy}{dx} = x \cdot \sin hx$

$$\frac{d^2y}{dx^2} = x \cdot \frac{d}{dx}(\sin hx) + \sin hx \cdot \frac{d}{dx}(x)$$
$\Rightarrow x \cdot \cos hx + \sin hx \cdot 1$
$\Rightarrow x \cos hx + \sin hx$

Question65

If $\left(x^2 - 3x + 2\right)^{\frac{y}{x^2-1}} = x + 2$, then $\left(\frac{dy}{dx}\right)_{x=0} =$

Options:

A.

2

B.

-2

C.

1

D.

-1

Answer: B

Solution:

Given equation

$\left(x^2 - 3x + 2\right) e^{\frac{y}{x-1}} = x + 2$

Differentiate both sides w.r.t., we get

$$\frac{d}{dx}\left(\left(x^2 - 3x + 2\right) \cdot e^{\frac{y}{x-1}}\right) = \frac{d}{dx}(x + 2)$$
$$\Rightarrow (2x - 3) e^{\frac{y}{x-1}} + \left(x^2 - 3x + 2\right) \cdot e^{\frac{y}{x-1}} \cdot \frac{d}{dx}\left(\frac{y}{x-1}\right) = 1$$
$$\Rightarrow (2x - 3) e^{\frac{y}{x-1}} + \left(x^2 - 3x + 2\right) \cdot e^{\frac{y}{x-1}} \left[\frac{\frac{dy}{dx}(x-1) - y}{(x-1)^2}\right] = 1 \qquad \ldots (i)$$

Now, substitute $x = 0$ into the given equation

$$\left(0^2-3(0)+2\right)e^{\frac{y}{(0-1)}}=0+2$$
$$\Rightarrow \quad 2e^{-y}=2 \Rightarrow e^{-y}=1$$
$$\Rightarrow \quad -y=\ln(1) \Rightarrow -y=0$$
$$\Rightarrow \quad y=0$$

Substitute $x=0$ and $y=0$ in Eq. (i), we get

$$(2(0)-3)e^{\frac{0}{0-1}}+\left(0^2-3(0)+2\right)e^{\frac{0}{0-1}}$$
$$\cdot\left[\frac{\frac{dy}{dx}(0-1)-0}{(0-1)^2}\right]=1$$
$$\Rightarrow -3e^0+2e^0\left(\frac{-dy}{dx}\right)=1$$
$$\Rightarrow \quad -3+2\left(\frac{-dy}{dx}\right)=1$$
$$\Rightarrow \quad -2\frac{dy}{dx}=1+3=4 \Rightarrow \frac{dy}{dx}=\frac{4}{-2}=-2$$
$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0}=-2$$

Question66

If $x=\frac{t^2}{1+t^5}, y=\frac{2t^3}{1+t^5}$ and $t\neq -1$ is a perimeter, then $\frac{dy}{dx}=$

Options:

A.

$\frac{2\left(3+2t^5\right)}{\left(2-3t^5\right)}$

B.

$\frac{2t\left(3-2t^5\right)}{\left(2-3t^5\right)}$

C.

$\frac{2t\left(3-2t^5\right)}{\left(2+3t^5\right)}$

D.

$\frac{2\left(3+2t^5\right)}{\left(2+3t^5\right)}$

Answer: B

Solution:

Given $x=\frac{t^2}{1+t^3}, y=\frac{2t^3}{1+t^5}$ and

$t \neq -1$

So, $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Now, $\frac{dx}{dt}=\frac{(2t)(1+t^5)-t^2 \cdot 5t^4}{(1+t^5)^2}$

$=\frac{2t-3t^6}{(1+t^5)^2}$

And, $\frac{dy}{dt}=\frac{(6t^2)(1+t^5)-(2t^3)(5t^4)}{(1+t^5)^2}$

$=\frac{6t^2-4t^7}{(1+t^5)^2}$

$$So,\ \frac{dy}{dx}=\frac{\left[\frac{6t^2-4t^7}{(1+t^5)^2}\right]}{\left[\frac{(2t-3t^6)}{(1+t^5)^2}\right]}=\frac{6t^2-4t^7}{2t-3t^6}$$
$$=\frac{2t^2(3-2t^5)}{t(2-3t^5)}=\frac{2t(3-2t^5)}{(2-3t^5)}$$

Question67

The acute angle between the curves $y=3x^2-2x-1$ and $y=x^3-1$ at their point of intersection which lies in the first quadrant is

Options:

A.

$\tan^{-1}\left(\frac{2}{121}\right)$

B.

$\tan^{-1}(2)$

C.

$\tan^{-1}\left(\frac{1}{13}\right)$

D.

$\frac{\pi}{2}$

Answer: A

Solution:

Set the equations equal to each other

$$\begin{aligned} & 3x^2 - 2x - 1 = x^3 - 1 \\ \Rightarrow \quad & x^3 - 3x^2 + 2x = 0 \\ \Rightarrow \quad & x(x-1)(x-2) = 0 \\ \Rightarrow \quad & x = 0, x = 1 \text{ and } x = 2 \end{aligned}$$

For $x = 0, y = 0^3 - 1 = -1$, Point : $(0, -1)$

For $x = 1, y = 1^3 - 1 = 0$, Point : $(1, 0)$

For $x = 2, y = 2^3 - 1 = 7$, Point : $(2, 7)$

So, the point of intersection in the first quadrant is $(2, 7)$.

For, $y_1 = 3x^2 - 2x - 1$

So, $m_1 = \frac{dy_1}{dx} = 6x - 2$

At $(2, 7), m_1 = 6(2) - 2 = 12 - 2 = 10$

For $y_2 = x^3 - 1, m_2 = \frac{dy_2}{dx} = 3x^2$

At $(2, 7), m_2 = 3\left(2^2\right) = 12$

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \left|\frac{10 - 12}{1 + (10)(12)}\right|$

$\Rightarrow \quad \left|\frac{-2}{1 + 120}\right| \Rightarrow \left|\frac{-2}{121}\right|$

$\Rightarrow \tan\theta = \frac{2}{121} \Rightarrow \theta = \tan^{-1}\left(\frac{2}{121}\right)$

Thus, the acute angle between the curves at their point of intersection in the first quadrant is $\tan^{-1}\left(\frac{2}{121}\right)$.

Question68

If the rate of change of the slope of the tangent drawn to the curve $y = x^3 - 2x^2 + 3x - 2$ at the point $(2, 4)$ is k times the rate of change of its abscissa, then $k =$

Options:

A.

2

B.

4

C.

6

D.

8

Answer: D

Solution:

Given, curve $y = x^3 - 2x^2 + 3x - 2$

So, slope of tangent, $m = \frac{dy}{dx}$

$= 3x^2 - 4x + 3$

Now, the rate of change of the slope of tangent $=$ second derivative $\frac{d^2y}{dx^2}$

$\Rightarrow \quad \frac{d}{dx}\left(3x^2 - 4x + 3\right) = 6x - 4$

At point $(2, 4)$,

$\frac{d^2y}{dx^2} = 6 \times 2 - 4 = 8$

Now, the abscissa is x. So, the rate of change of the abscissa is $\frac{dx}{dt}$

Now, given $\frac{d}{dt}\left(\frac{dy}{dx}\right) = k\frac{dx}{dt}$

$\Rightarrow \quad \frac{d^2y}{dx^2} \cdot \frac{dx}{dt} = k\frac{dx}{dt} \Rightarrow \frac{d^2y}{dx^2} = k$

$\Rightarrow \quad k = 8 \quad$ [using Eq. (i)]

Question69

If $1^\circ = 0.0175$ radians, then the approximate value of $\sec 58^\circ$ is

Options:

A.

1.9899

B.

1.8788

C.

1.8511

D.

1.9677

Answer: B

Solution:

Let $f(x)=\sec(x)$

Since 58° is close to 60° and $\sec\left(60^{\circ}\right)=2$

Also, $60^{\circ}=60\times 0.0175$ radians $=1.05$

Now, $f(x)=\sec x$

$$\begin{aligned} \Rightarrow \quad f'(x) &= \sec(x)\tan(x) \\ f'\left(60^{\circ}\right) &= \sec\left(60^{\circ}\right)\tan\left(60^{\circ}\right) \\ &= 2\times\sqrt{3}=2\sqrt{3} \end{aligned}$$

And, change in angle,

$$\begin{aligned} \Delta x &= 58^{\circ}-68^{\circ}=-2^{\circ} \\ &= -2\times 0.0175 \text{ radians} \\ &= -0.035 \text{ radians} \end{aligned}$$

Now, $\Delta y\approx f'(x)\cdot\Delta x$

$$\begin{aligned} &\approx 2\sqrt{3}\times(-0.035) \\ &\approx 2\times 1.732\times(-0.035) \\ &\approx -0.12124 \end{aligned}$$

So, $\sec\left(58^{\circ}\right)\approx f\left(60^{\circ}\right)+\Delta y$

$\Rightarrow \quad 2-0.12124\approx 1.87876$

So, the approximate value of $\sec\left(58^{\circ}\right)$ is 1.8788 .

Question70

If $f(x) = x + \log\left(\frac{x-1}{x+1}\right)$ **is a well-defined real valued function, then** f **is**

Options:

A.

monotonically decreasing function

B.

monotonically increasing function

C.

increasing in $(1, \infty)$ and decreasing in $(-\infty, -1)$

D.

decreasing in $(1, \infty)$ and increasing in $(-\infty, -1)$

Answer: B

Solution:

Given, $f(x) = x + \log\left(\frac{x-1}{x+1}\right)$

For, $\log\left(\frac{x-1}{x+1}\right)$ to be well-defined, $\frac{x-1}{x+1} > 0$.

This inequality holds when

$\Rightarrow$ Both $x - 1 > 0$ and $x + 1 > 0 \Rightarrow x > 1$

$\Rightarrow$ Both $x - 1 < 0$ and $x + 1 < 0 \Rightarrow x < -1$

So, the domain of $f(x)$ is $(-\infty, -1) \cup (1, \infty)$

Now, $f'(x) = \frac{d}{dx}\left[x + \log\left(\frac{x-1}{x+1}\right)\right]$

$\Rightarrow 1 + \frac{(x+1)}{(x-1)} \cdot \frac{(x+1) \cdot 1 - 1 \cdot (x-1)}{(x+1)^2}$

$$\Rightarrow 1 + \frac{(x+1) - (x-1)}{(x-1)(x+1)}$$

$$\Rightarrow 1 + \frac{2}{(x-1)(x+1)}$$

Since, $x \in (-\infty, -1) \cup (1, \infty)$, we have $x^2 > 1$

$\Rightarrow x^2 - 1 > 0$

Thus, $\frac{2}{x^2-1} > 0$

So, $f'(x) = 1 + \frac{2}{x^2-1} > 1 + 0 = 1$

Since, $f'(x) > 0$ for all x in the domain, the function $f(x)$ is monotonically increasing.

Question71

A real valued function $f(x) = \left|x^2 - 3x + 2\right| + 2x - 3$ is defined on $[-2, 1]$. If m and M are absolute minimum and absolute maximum values of f respectively, then $M - 4m =$

Options:

A.

0

B.

1

C.

15

D.

10

Answer: D

Solution:

Given, $f(x) = |x^2 - 3x + 2| + 2x - 3$ is defined on $[-2, 1]$

$x^2 - 3x + 2 = (x-1)(x-2)$

So, on the interval $[-2, 1]$

If $x \leq 1$, then $x - 1 \leq 0$

If $x \leq 2$ then $x - 2 \leq 0$

$\therefore$ On $[-2, 1], (x-1)(x-2) \geq 0$

So, $\left|x^2 - 3x + 2\right| = x^2 - 3x + 2$ for $x \in [-2, 1]$

$$\therefore f(x) = \left(x^2 - 3x + 2\right) + 2x - 3$$
$$= x^2 - x - 1$$

Now, $f'(x) = 2x - 1$

For, critical points, set $f'(x) = 0$ we get

$$\begin{aligned} & 2x - 1 = 0 \\ \Rightarrow \quad & x = \frac{1}{2} \in [-2, 1] \end{aligned}$$

Now, at $x = -2 f(-2) = (-2)^2 - (-2) - 1$

$\Rightarrow \quad 4 + 2 - 1 = 5$

At $\quad x = \frac{1}{2}, f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 1$

$\Rightarrow \quad \frac{1}{4} - \frac{1}{2} - 1 \Rightarrow \frac{1-2-4}{4} = -\frac{5}{4}$

At $\quad x = 1, f(1) = 1^2 - 1 - 1 = -1$

So, the absolute minimum value, $m = \frac{-5}{4}$ and the absolute maximum value, $M = 5$

$$\begin{aligned} & \therefore M - 4m = 5 - 4 \times \left(-\frac{5}{4}\right) \\ & \Rightarrow \quad 5 + 5 = 10 \end{aligned}$$

Question72

$\int \frac{2 \sin x - 3 \cos x}{4 \cos x - 3 \sin x} dx =$

Options:

A.

$\frac{1}{25}[17 \log |4 \cos x - 3 \sin x| - 6x] + C$

B.

$\frac{1}{25}[x - 18 \log |4 \cos x - 3 \sin x|] + C$

C.

$\frac{1}{25}[\log |4 \cos x - 3 \sin x| - 18x] + C$

D.

$\frac{1}{25}[17x - 6 \log |4 \cos x - 3 \sin x|] + C$

Answer: C

Solution:

$I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$

Let $t = 4\cos x - 3\sin x$

$\Rightarrow \frac{dt}{dx} = -4\sin x - 3\cos x$

Now, $2\sin x - 3\cos x$

$$\Rightarrow A(-4\sin x - 3\cos x) + B(4\cos x - 3\sin x)$$
$$\Rightarrow (-4A - 3B)\sin x + (-3A + 4B)\cos x$$

Comparing the coefficients, we get

$-4A - 3B = 2 \text{ and } -3A + 4B = -3$

Solving these two equations, we get

$A = \frac{1}{25} \text{ and } B = \frac{-18}{25}$

$$\text{So, } I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$$
$$\Rightarrow \frac{1}{25} \int \frac{-4\sin x - 3\cos x}{4\cos x - 3\sin x} dx + \left(\frac{-18}{25}\right) \int \frac{4\cos x - 3\sin x}{4\cos x - 3\sin x} dx$$
$$\Rightarrow \frac{1}{25} \int \frac{dt}{t} - \frac{18}{25} \int dx \Rightarrow \frac{1}{25} \log|t| - \frac{18}{25} x + C$$
$$\Rightarrow \frac{1}{25} \log|4\cos x - 3\sin x| - \frac{18}{25} x + C$$
$$= \frac{1}{25} [\log|4\cos x - 3\sin x| - 18x] + C$$

Question73

$$\int e^{4x}(\sin 3x - \cos 3x)dx =$$

Options:

A.

$\frac{e^{4x}}{25}(7\sin 3x - \cos 3x) + C$

B.

$\frac{e^{4x}}{25}(\sin 3x - 7\cos 3x) + C$

C.

$\frac{e^{4x}}{5}(7\sin 3x + \cos 3x) + C$

D.

$\frac{e^{4x}}{5}(\sin 3x + 7\cos 3x) + C$

Answer: B

Solution:

$$\int e^{4x}(\sin 3x - \cos 3x)dx$$

$$= \int e^{4x}\sin 3x dx - \int e^{4x}\cos 3x dx$$

Using the standard integrals

$$\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

Here, $a = 4, b = 3$

$$\text{So, } \int e^{4x}(\sin 3x - \cos 3x)dx$$
$$= \frac{e^{4x}}{4^2+3^2}(4\sin 3x - 3\cos 3x) - \frac{e^{4x}}{4^2+3^2}(4\cos 3x + 3\sin 3x) + C$$
$$\Rightarrow \frac{e^{4x}}{25}[4\sin 3x - 3\cos 3x - 4\cos 3x - 3\sin 3x] + C$$
$$\Rightarrow \frac{e^{4x}}{25}[\sin 3x - 7\cos 3x] + C$$

Question74

$$\int \left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx =$$

Options:

A.

$\frac{1}{1+(\log x)^2} + C$

B.

$\frac{\log x}{1+(\log x)^2} + C$

C.

$\frac{x}{1+(\log x)^2} + C$

D.

$\frac{x^2}{1+(\log x)^2}+C$

Answer: C

Solution:

$I=\int\left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx$

Let $y=\log x$. Then, $x=e^y \Rightarrow dx=e^y \cdot dy$

So, $I=\int\left(\frac{1-y}{1+y^2}\right)^2 e^y dy$

$$=\int\left(\frac{1+y^2-2y}{(1+y^2)^2}\right)e^y dy$$

$$\Rightarrow \quad \int\left(\frac{1}{1+y^2}-\frac{2y}{(1+y^2)^2}\right)e^y dy$$

Since, $\int(f(y)+f'(y))e^y dy=f(y)e^y+C$

Here, $f(y)=\frac{1}{1+y^2}$ and $f'(y)=\frac{-2y}{(1+y^2)^2}$

$$\therefore \quad I=f(y)e^y+C=\frac{1}{1+y^2}e^y+C$$

$$=\frac{1}{1+(\log x)^2}\cdot e^{\log x}+C$$

$$=\frac{x}{1+(\log x)^2}+C$$

Question75

If $\int(x+2)\sqrt{x^2-x+2}dx=\frac{1}{3}f(x)+\frac{5}{8}g(x)+\frac{35}{16}h(x)+C$ then $f(-1)+g(-1)+h\left(\frac{1}{2}\right)=$

Options:

A.

-4

B.

$2+\ln\left(\frac{\sqrt{7}}{2}\right)$

C.

4

D.

-2

Answer: B

Solution:

$$\text{Given, } \int (x+2)\sqrt{x^2-x+2}dx$$
$$\Rightarrow \quad \frac{1}{3}f(x)+\frac{5}{8}g(x)+\frac{35}{16}h(x)+C$$

Let $x+2=A(2x-1)+B$

Comparing coefficients, we get

$$2A=1$$
$$\Rightarrow \quad A=\frac{1}{2}, -A+B=2$$
$$\Rightarrow \quad B=2+A=2+\frac{1}{2}=\frac{5}{2}$$
$$\text{So, } x+2=\frac{1}{2}(2x-1)+\frac{5}{2}$$
$$\int (x+2)\sqrt{x^2-x+2}dx$$
$$=\int\left(\frac{1}{2}(2x-1)+\frac{5}{2}\right)\sqrt{x^2-x+2}dx$$
$$=\frac{1}{2}\int(2x-1)\sqrt{x^2-x+2}dx+\frac{5}{2}\int\sqrt{x^2-x+2}dx$$

Let $u=x^2-x+2$, then $du=(2x-1)dx$

$$=\frac{1}{2}\int\sqrt{u}du+\frac{5}{2}\int\sqrt{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}dx$$
$$=\frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{5}{2}\left[\frac{2x-1}{4}\sqrt{x^2-x+2}+\frac{7}{8}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|\right]+c$$
$$=\frac{1}{3}\left(x^2-x+2\right)^{\frac{3}{2}}+\frac{5(2x-1)}{8}\sqrt{x^2-x+2}+\frac{35}{16}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|+c$$

Comparing with original equation, we get

$$f(x)=\left(x^2-x+2\right)^{\frac{3}{2}}$$
$$g(x)=(2x-1)\sqrt{x^2-x+2}$$
$$h(x)=\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|$$

So, $f(-1)=\left[(-1)^2-(-1)+2\right]^{\frac{3}{2}}$

$=[1+1+2]^{\frac{3}{2}}=4^{\frac{3}{2}}=8$

$g(-1)=(2(-1)-1)\sqrt{(-1)^2-(-1)+2}$

$=-3\sqrt{1+1+2}=-3\sqrt{4}$

$=-3\times 2=-6$

$$h\left(\frac{1}{2}\right)=\ln\left|\frac{1}{2}-\frac{1}{2}+\sqrt{\left(\frac{1}{2}\right)^2-\frac{1}{2}+2}\right|$$

$$\Rightarrow \ln\left|\,0+\sqrt{\frac{1}{4}-\frac{1}{2}+2}\right|$$

$$\Rightarrow \ln\left|\sqrt{\frac{1-2+8}{4}}\right| \Rightarrow \ln\left|\sqrt{\frac{7}{4}}\right|=\ln\left(\frac{\sqrt{7}}{2}\right)$$

$$\text{So, } f(-1)+g(-1)+h\left(\frac{1}{2}\right)=8-6+\ln\left(\frac{\sqrt{7}}{2}\right)$$

$$=2+\ln\left(\frac{\sqrt{7}}{2}\right)$$

Question76

$$\int_0^2 \sqrt{(x+3)(2-x)}dx=$$

Options:

A.

$\frac{25}{8}\cos^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$

B.

$\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$

C.

$\frac{\pi}{2}$

D.

π

Answer: A

Solution:

$I=\int_0^2 \sqrt{(x+3)(2-x)}dx$

$$\Rightarrow \quad \int_0^2 \sqrt{-x^2-x+6dx}$$

$$\Rightarrow \quad \int_0^2 \sqrt{-\left(x^2+x-6\right)dx}$$

$$\Rightarrow \quad \int_0^2 \sqrt{\frac{25}{4}-\left(x+\frac{1}{2}\right)^2} dx$$

Put $u=x+\frac{1}{2} \Rightarrow du=dx$

When $x=0, u=\frac{1}{2}$ and $x=2$

$$u=2+\frac{1}{2}=\frac{5}{2}$$

$$I=\int_{\frac{1}{2}}^{\frac{5}{2}} \sqrt{\left(\frac{5}{2}\right)^2-u^2du}$$

$$\Rightarrow \quad \left[\frac{u}{2} \sqrt{\frac{25}{4}-u^2}+\frac{\frac{25}{4}}{2} \sin^{-1}\left(\frac{u}{\frac{5}{2}}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$

$$\Rightarrow \quad \left[\frac{u}{2} \sqrt{\frac{25}{4}-u^2}+\frac{25}{8} \sin^{-1}\left(\frac{2u}{5}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$

$$\Rightarrow \quad \left\{\frac{\frac{5}{2}}{2} \sqrt{\frac{25}{4}-\frac{25}{4}}+\frac{25}{8} \sin^{-1}(1)\right\}-\left\{\frac{1}{4} \sqrt{\frac{25}{4}-\frac{1}{4}}+\frac{25}{8} \sin^{-1}\left(\frac{1}{5}\right)\right\}$$

$$\Rightarrow \quad 0+\frac{25}{8} \cdot \frac{\pi}{2}-\frac{1}{4} \sqrt{6}-\frac{25}{8} \sin^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \quad \frac{25\pi}{16}-\frac{\sqrt{6}}{4}-\frac{25}{8} \sin^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \quad \frac{25}{8}\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{5}\right)\right)-\frac{\sqrt{6}}{4}$$

$$\Rightarrow \quad \frac{25}{8} \cos^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$$

Question77

$\int_0^{\pi/4} x^2 \sin 2x dx$

Options:

A.

$\frac{\pi^2-2}{8}$

B.

$\frac{\pi(\pi-2)}{8}$

C.

$\frac{\pi-2}{8}$

D.

$\frac{\pi+2}{8}$

Answer: C

Solution:

$\int_0^{\frac{\pi}{4}} x^2 \sin 2x dx, I = \int x^2 \sin 2x dx$

Put $t = 2x \Rightarrow dt = 2 \cdot dx$

So, $I = \int \frac{t^2 \sin t}{8} dt = \frac{1}{8} \int t^2 \sin t dt$

$$= \frac{1}{8}\left[t^2 \cdot (-\cos t) - 1 \cdot (-2) \cdot \int t \cos t dt\right]$$

$$= \frac{1}{8}\left[-t^2 \cos t + 2 \int t \cdot \cos t dt\right]$$

$$= \frac{1}{8}\left[-t^2 \cos t + 2(t \cdot \sin t - (-\cos t))\right]$$

$$= \frac{1}{8}\left[-t^2 \cos t + 2t \sin t + 2 \cos t\right]$$

$$= \frac{1}{8}\left[(2x)^2 \cos(2x) + 2(2x \cdot \sin(2x)) + 2 \cos(2x)\right]$$

$$= \frac{-x^2 \cos(2x) + x \sin(2x)}{2} + \frac{\cos(2x)}{4}$$

So, $\int_0^{\frac{\pi}{4}} x^2 \sin 2x dx$

$$= \left[\frac{-x^2\cos(2x) + x\sin(2x)}{2} + \frac{\cos(2x)}{4}\right]_0^{\frac{\pi}{4}}$$

$$= \left\{\frac{-\left(\frac{\pi}{4}\right)^2\cos\left(2\times\frac{\pi}{4}\right) + \frac{\pi}{4}\sin\left(2\times\frac{\pi}{4}\right)}{2} + \frac{\cos\left(\frac{2\pi}{4}\right)}{4}\right\} - \left\{\frac{0}{2} + \frac{\cos(0)}{4}\right\}$$

$$\Rightarrow \left(0 + \frac{\pi}{8}\cdot 1 + \frac{1}{4}\cdot 0\right) - \left(0 + \frac{1}{4}\right)$$

$$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

Question78

$\int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx =$

Options:

A.

$\frac{3\pi}{128}$

B.

$\frac{9\pi}{32}$

C.

$\frac{9\pi}{64}$

D.

$\frac{3\pi}{64}$

Answer: D

Solution:

$$\int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx$$

$$f(x) = \sin^4 x \cos^6 x$$

Since, $\sin x$ is an odd function and $\sin^2 x$ is an even function and $\cos^6 x$ is an even function.

So, $f(x)$ is an even function.

$$\therefore \int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx = 2\int_0^{2\pi} \sin^4 x \cos^6 x dx$$

$$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx\right]$$

$$= 2 \times 2 \int_0^{\pi} \sin^4 x \cos^6 x dx$$

$$\left[\because \int_0^{n\pi} f(x)dx = n\int_0^T f(x)dx\right]$$

$$= 4 \int_0^{\pi} \sin^4 x \cos^6 x dx$$

$$= 4 \times 2 \int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x dx$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx\right]$$

$$= 8 \int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x dx$$

[$\because$ If m and n are both even, then

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx =$$

$$\left.\frac{[(m-1)(m-3)\ldots 2 \text{ or } 1][(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1]} \times \frac{\pi}{2}\right]$$

$$= 8 \times \frac{3 \times 1 \times 5 \times 3 \times 1}{10 \times 8 \times 6 \times 4 \times 2} \times \frac{\pi}{2}$$

$$= \frac{3\pi}{64}$$

Question79

If $\cos x \frac{dy}{dx} = y \sin x - 1, x \neq (2n+1)\frac{\pi}{2}, n \in Z$ is the differential equation corresponding to the curve $y = f(x)$ and $f(0) = 1$, then $f(x)$

Options:

A.

$(1-x)\sec x$

B.

$(1-x)\cos x$

C.

$x+\cos x$

D.

$x+\sec x$

Answer: A

Solution:

Given, $\cos x \frac{dy}{dx}=y \sin x-1$,

$$
\begin{aligned}
& x \neq(2 n+1) \frac{\pi}{2}, n \in Z \\
\Rightarrow \quad & \frac{dy}{dx}=\frac{y \sin x}{\cos x}-\frac{1}{\cos x} \\
& =y \tan x-\sec x \\
\Rightarrow \quad & \frac{dy}{dx}-y \tan x=-\sec x
\end{aligned}
$$

This is in the form $\frac{dy}{dx}+P(x) y=Q(x)$,

where, $P(x)=-\tan x$ and $Q(x)=-\sec x$

$$
\begin{aligned}
\mathrm{IF} & =e^{\int P(x) dx}=e^{\int-\tan x dx} \\
& =e^{\ln |\cos x|}=|\cos x|
\end{aligned}
$$

So, the general solution is

$$
\begin{aligned}
y \cdot \text { IF } & =\int A(x) \cdot \text { IF } dx+C \\
y \cdot \cos x & =\int-\sec x \cdot \cos x dx+C \\
& =-\int dx+C=-x+C \\
& =y=\frac{-x+C}{\cos x}
\end{aligned}
$$

Given, $f(0)=1$

$\Rightarrow \quad y=1$ and $x=0$

So, $1=\frac{-0+C}{\cos (0)}$

$\Rightarrow 1=C$

So, $y=\frac{-x+1}{\cos x}=(1-x) \sec x$

Question80

The general solution of the differential equation $2dx + dy = (6xy + 4x - 3y)dx$ is

Options:

A.

$2\log|2x-1| = 3y^2 + 4y + C$

B.

$\log|3y+2| = 3x^2 - 3x + C$

C.

$\log|3y+2| = x^2 - x + C$

D.

$\log|2x-1| = 3y^2 - 4y + C$

Answer: B

Solution:

Given, differential equation

$$
\begin{aligned}
& 2dx + dy = (6xy + 4x - 3y)dx \\
\Rightarrow \quad & dy = (6xy + 4x - 3y - 2)dx \\
\Rightarrow \quad & \frac{dy}{dx} = 6xy + 4x - 3y - 2 \\
\Rightarrow \quad & \frac{dy}{dx} = 6xy + 4x - 2 - 3y \\
\Rightarrow \quad & 2x(3y+2) - 1(3y+2) \\
\Rightarrow \quad & (3y+2)(2x-1) \\
\Rightarrow \quad & \frac{dy}{3y+2} = dx(2x-1)
\end{aligned}
$$

Integrating to both sides w.r.t x, we get

$$
\begin{aligned}
& \int \frac{dy}{3y+2} = \int (2x-1)dx \\
\Rightarrow & \frac{1}{3}\log|3y+2| = \frac{2x^2}{2} - x + C_1 \\
\Rightarrow & \frac{1}{3}\log|3y+2| = x^2 - x + C_1 \\
\Rightarrow & \quad \log|3y+2| = 3x^2 - 3x + 3C_1 \\
& = 3x^2 - 3x + C
\end{aligned}
$$

(where $C = 3C_1$)

So, $\log|3y+2| = 3x^2 - 3x + C$

Chemistry

Question1

The radius of stationary state $(n=2)$ of hydrogen atom is $x$pm. The radius of stationary state $(n=3)$ of He^+ion (in pm) is

Options:

A.

$9/8x$

B.

$9x/8$

C.

$16x/9$

D.

$9/16x$

Answer: B

Solution:

Radius of nth orbit of hydrogen and hydrogen like atom, $r_n = \frac{n^2 a_0}{Z}$

For $n = 2, r_n = x\text{pm}, Z = 1$ (for H-atom)

$x = \frac{2^2(a_0)}{1} \Rightarrow a_0 = \frac{x}{4}$ $\ldots(i)$

For $n = 3, Z = 2, r_n = ?$

(for He^+atom)

$$r_n = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$
$$\Rightarrow r_n = \frac{9 \times x}{2 \times 4} = \frac{9x}{8}$$

Question2

When electromagnetic radiation of wavelength 310 nm falls on the surface of a metal having work function 3.55 eV , the velocity of photoelectrons emitted is $x \times 10^5 \mathrm{~ms}^{-1}$. The value of x is (Nearest integer) $\left(m_e=9 \times 10^{-31} \mathrm{~kg}\right)$

Options:

A.

2

B.

4

C.

5

D.

6

Answer: B

Solution:

Given, $\lambda=310 \mathrm{~nm}$

$=310 \times 10^{-9} \mathrm{~m}$
$\phi=3.55 \mathrm{eV} \Rightarrow v=x \times 10^5 \mathrm{~m/s}$
$x=?$

Energy of incident photon,

$$E=\frac{h c}{\lambda}=\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{310 \times 10^{-9}}$$
$$E=6.412 \times 10^{-19} \mathrm{~J}$$

Kinetic energy of photoelectrons,

$\mathrm{KE}=E-\phi$

$\mathrm{KE}=6.412 \times 10^{-19}-3.55 \times 1.6 \times 10^{-19}$
$\mathrm{KE}=0.732 \times 10^{-19} \mathrm{~J}$

Velocity of photoelectrons,

$$v=\sqrt{\frac{2\mathrm{KE}}{m_e}}$$

$$=\sqrt{\frac{2\times 0.732\times 10^{-19}}{9\times 10^{-31}}}$$

$$v=4\times 10^5\ \mathrm{m/s}$$

or $\quad x=4$

Question3

In which of the following options, elements are correctly arranged in the increasing order of their atomic radius?

Options:

A.

$\mathrm{Si}<\mathrm{P}<\mathrm{Na}<\mathrm{N}<\mathrm{F}$

B.

$\mathrm{Na}<\mathrm{Si}<\mathrm{P}<\mathrm{N}<\mathrm{F}$

C.

$\mathrm{F}<\mathrm{N}<\mathrm{P}<\mathrm{Si}<\mathrm{Na}$

D.

$\mathrm{N}<\mathrm{F}<\mathrm{Si}<\mathrm{P}<\mathrm{Na}$

Answer: C

Solution:

The correct order of increasing atomic radius is $\mathrm{F}<\mathrm{N}<\mathrm{P}<\mathrm{Si}<\mathrm{Na}$.

It is because atomic radius decreases across a period. While it increases down the group.

Question4

A, BC, D and E are elements with atomic numbers 13 , $11, 9, 7$ and 16 respectively. Among these elements, ion of an element X has

largest size and ion of an element Y has smallest size. X and Y are respectively.(Assume that all ions have nearest inert gas configuration)

Options:

A.

D, A

B.

A, D

C.

E, A

D.

D, A

Answer: C

Solution:

(i) Elements with atomic number $13, 11, 9, 7$ and 16 are aluminium, sodium, fluorine, nitrogen and sulphur respectively. Thus, largest ions is S^{2-}, sulphide ion while smallest ion will be Al^{3+} ion. So, elements X and Y are E and A respectively.

Question5

Identify the pair of molecules in which the hybridisation of the central atom is sp^2 with bent geometry.

Options:

A.

H_2O, SO_2

B.

SO_2, O_3

C.

H_2O, O_3

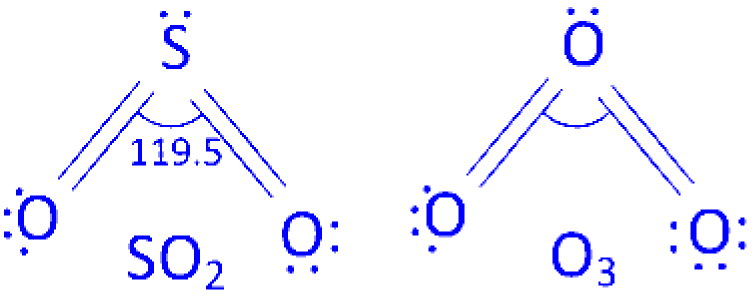
D.

N_2O, H_2O

Answer: B

Solution:

SO_2 and O_3 has sp^2 hybridisation with bent geometry.



Question6

Consider the following statements

I. In the conversion of O_2 to O_2^{2+} bond order decreases.

II. In the conversion of O_2 to O_2^{2+} magnetic property is not changed.

III. In the conversion of O_2 to O_2^{2+} bond length decreases.

IV. O_2^{2-} and B_2 have same bond order.

Identify the correct statements

Options:

A.

I and III only

B.

III and IV only

C.

II and III only

D.

I and IV only

Answer: C

Solution:

Statement given in III and IV are correct. While the statements given in I and II are incorrect. Their correct forms are

I. When O_2 is converted to O_2^{2+}, the bond order increases from 2 to 3. II. In conversion of O_2 to O_2^{2+}, the magnetic property changes from paramagnetic to diamagnetic.

Question7

The RMS velocity of dihydrogen is $\sqrt{7}$ times more than that of dinitrogen. If T_{H_2} and T_{N_2} are the temperatures of dihydrogen and dinitrogen, then the correct relationship between them is

Options:

A.

$T_{H_2} = T_{N_2}$

B.

$T_{H_2} > T_{N_2}$

C.

$T_{H_2} = \sqrt{7} T_{N_2}$

D.

$T_{H_2} = \frac{T_{N_2}}{2}$

Answer: D

Solution:

v_{rms} is given by, $v_{rms} = \sqrt{\frac{3RT}{M}}$

Also, it is given that

$$v_{rms(H_2)} = \sqrt{7} v_{rms(N_2)}$$
$$\sqrt{\frac{3RT_{H_2}}{M_{H_2}}} = \sqrt{7} \times \sqrt{\frac{3RT_{N_2}}{M_{N_2}}}$$
$$\sqrt{\frac{3T_{H_2}}{2}} = \sqrt{7} \times \sqrt{\frac{3T_{N_2}}{28}}$$
$$\frac{T_{H_2}}{2} = \frac{T_{N_2}}{4} \Rightarrow T_{H_2} = \frac{T_{N_2}}{2}$$

Question8

Which of the following solution has highest amount of solute?

Options:

A.

1.0 L of $0.25 M Na_2CO_3 (106u)$

B.

0.25 L of $0.2 M Na_2SO_4 (142u)$

C.

0.5 L of $1.0 M KMnO_4 (158u)$

D.

0.75 L of $0.5 M (NH_2)_2CO (60u)$

Answer: C

Solution:

Mass of solute $=$ Molarity $\times$ Volume (L)$\times$ Molar mass

(i) $0.25 \times 1 \times 106 = 26.5$ g

(ii) $0.2 \times 0.25 \times 142 = 7.1$ g

(iii) $1 \times 0.5 \times 158 = 79$ g

(iv) $0.5 \times 0.75 \times 60 = 22.5$ g

Hence, 0.5 L of $1.0 M KMnO_4$ has highest amount of solute.

Question9

At 298 K , the enthalpy change (in kJ) for the reaction given below is

$$CH_4(g)+O_2(g)\longrightarrow C(s)+2H_2O(l)$$

$$\begin{aligned}Given: H_2(g)+\frac{1}{2}O_2(g) &\longrightarrow H_2O(l);\Delta H^{\ominus}=-286\text{ kJ}\\ C(s)+O_2(g) &\longrightarrow CO_2(g);\Delta H^{\ominus}=-394\text{ kJ}\\ CH_4(g)+2O_2(g) &\longrightarrow CO_2(g)+2H_2O(l)\Delta H^{\ominus}=-890\text{ kJ}\end{aligned}$$

Options:

A.

+496

B.

-496

C.

-1284

D.

+680

Answer: B

Solution:

$Given,\ rH_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(l);\Delta H^{\circ}=-286\text{ kJ}\quad\ldots(\text{i})$

$C(s)+O_2(g)\longrightarrow CO_2(g);\Delta H^{\circ}=-394\text{ kJ}\quad\ldots(ii)$

$CH_4(g)+2O_2(g)\longrightarrow CO_2(g)+2H_2O(l);\Delta H^{\circ}=-890\text{ kJ}\quad\ldots(iii)$

Reverse reaction given in Eq. (ii),
$CO_2(\text{ g})\longrightarrow C(\text{ s})+O_2(g);\Delta H^{\circ}=+394\text{ kJ}\quad\ldots(\text{iv})$

Now, add Eq. (iii) and (iv),

$$CH_4(g)+2O_2(g)+CO_2(g)\longrightarrow CO_2(s)+2H_2O(l)+C(s)+O_2(g)$$
$$CH_4(g)+O_2(g)\longrightarrow C(s)+2H_2O(l)$$
$$\text{So, }\Delta H^\circ=-890+394=-496\text{ kJ}$$

Question10

For the reaction $N_2O_4(g)\rightleftharpoons 2NO_2(g)$, the correct relation between degree of dissociation (α) of $N_2O_4(g)$ and equilibrium constant, K_p is ($p=$ total pressure of mixture)

Options:

A.

$\alpha=\frac{K_p/p}{4+\frac{K_p}{p}}$

B.

$\alpha=\frac{K_p}{4+K_p}$

C.

$\alpha=\left(\frac{K_p/p}{4+\frac{K_p}{p}}\right)^{\frac{1}{2}}$

D.

$\alpha=\left(\frac{K_p}{4+K_p}\right)^{\frac{1}{2}}$

Answer: C

Solution:

Given, reaction is

	$N_2O_4(g)$ $\rightleftharpoons$	$2NO_2(g)$
Initial moles	1	0
Degree of dissociation=	α	
At equilibrium	1-α	2α

Partial pressure is given by

$$p_{N_2O_4} = \left(\frac{1-\alpha}{1+\alpha}\right)p; \quad p_{NO_2} = \frac{2\alpha}{1+\alpha} \cdot p$$

$$K_p = \frac{(p_{NO_2})^2}{p_{N_2O_4}}$$

Substituting the values,

$$\alpha = \left(\frac{\frac{K_p}{p}}{4+\frac{K_p}{p}}\right)^{1/2}.$$

Question11

Oxidation state of hydrogen in compound X is -1 and in compound Y is $+1$. X and Y are respectively

Options:

A.

$LiAlH_4, H_2O$

B.

NH_3, NaH

C.

CH_4, H_2O

D.

$H_2S, NaBH_4$

Answer: A

Solution:

$LiAlH_4 = \text{Total charge} = 0$

Li oxidation number $= +1$

Al oxidation number $= +3$

$$1+3+x \times 4 = 0$$
$$x = -1$$

$H_2O = 2 \times x - 2 = 0, x = +1$

Hence, $X = LiAlH_4, Y = H_2O$

Question12

Match the following

Lis- I (Chemical)		List-II (Use)	
A.	KOH	I.	Coolant
B.	$Na(l)$	II.	Antacid
C.	Li	III.	Electrochemical cells
D.	$Mg(OH)_2$	IV	Absorbent for CO_2

The correct answer is

Options:

A.

A-II, B-III, C-IV, D-I

B.

A-IV, B-I, C-III, D-II

C.

A-IV, B-III, C-II, D-I

D.

A-III, B-IV, C-I, D-II

Answer: B

Solution:

A. KOH (Potassium hydroxide)

- KOH is a strong base that absorbs carbon dioxide (CO_2) forming potassium carbonate.

✅ **Use:** Absorbent for CO_2.

$\rightarrow A \rightarrow IV$

B. Na(l) (Liquid sodium)

- Liquid sodium metal is used as a **coolant** in nuclear reactors because of its high thermal conductivity.

✅ **Use:** Coolant.

→ B → I

C. Li (Lithium)

- Lithium is used in **electrochemical cells** (batteries).

✅ **Use:** Electrochemical cells.

→ C → III

D. $Mg(OH)_2$ (Magnesium hydroxide)

- $Mg(OH)_2$ is a weak base, used as an **antacid** ("milk of magnesia").

✅ **Use:** Antacid.

→ D → II

✅ **Therefore, correct matching is:**

A–IV, B–I, C–III, D–II

👉 **Correct Option: B**

Question13

When burnt in excess of oxygen, sodium forms a compound X and potassium forms a compound Y. The magnetic natures of X and Y respectively are

Options:

A.

Both X and Y are paramagnetic in nature

B.

X is diamagnetic and Y is paramagnetic in nature

C.

X is paramagnetic and Y is diamagnetic in nature

D.

Both X and Y are diamagnetic in nature

Answer: B

Solution:

The reaction mention is as follows,

(i) $2Na(s) + \underset{(excess)}{O_2(g)} \longrightarrow \underset{(X)}{Na_2O_2(s)}$

Sodium peroxide O_2^{2-} has all its electrons paired making it diamagnetic.

$$\text{(ii) } K(s) + O_2(g) \longrightarrow \underset{(y)}{KO_2(s)}$$

The superoxide has one unpaired electrons, making it paramagnetic.

Question14

The correct order of atomic radii of group 13 elements is

Options:

A.

Al > TI > Ga > In

B.

Al > Ga > In > TI

C.

TI > In > Ga > AI

D.

TI > In > AI > Ga

Answer: D

Solution:

The correct order of atomic radii in group 13 elements is, Tl > In > Al > Ga.

Usually radii increases down the group, due to addition of shell, but Ga has smaller radii than Al due to poor shielding effect of *d*-electrons.

Question15

Observe the following oxides. The number of amphoteric oxides from the given list is $CO, B_2O_3, SnO_2, PbO_2, Ga_2O_3, SnO, PbO, CO_2$

Options:

A.

3

B.

4

C.

5

D.

6

Answer: C

Solution:

Amphoteric oxide:

$SnO, PbO, SnO_2, PbO_2, Ga_2O_3$

Acidic oxide : CO_2, B_2O_3

Neutral : CO

Question16

Among the following compounds, which one is not primarily responsible for depletion of ozone layer in stratosphere?

Options:

A.

NO

B.

CF_2Cl_2

C.

CH_4

D.

Cl_2

Answer: C

Solution:

🌍 Background:

The **depletion of the ozone layer** is mainly caused by **halogen (Cl, Br)** and **nitrogen oxides (NO_x)** that catalytically destroy ozone (O_3) molecules in the stratosphere.

The main culprits are:

- **CFCs (chlorofluorocarbons)** such as CF_2Cl_2 (Freon-12).
- **NO and NO_2** (from supersonic aircrafts, etc.) to a limited extent.

🔷 Option A: NO

- Nitric oxide (**NO**) can indeed react with ozone and participate in catalytic cycles that destroy ozone.
- So **NO does contribute** to ozone layer depletion.

✅ *Primarily responsible.*

🔷 Option B: CF_2Cl_2 (CFC-12)

- A chlorofluorocarbon that releases chlorine radicals (Cl·) under UV light in the stratosphere.
- These radicals catalytically destroy ozone.
- CFCs are the **principal cause** of ozone layer depletion.

✅ *Primarily responsible.*

🔷 Option C: CH_4 (Methane)

- Methane does not destroy ozone directly.
- In fact, methane can **react with Cl· radicals**, forming HCl and thereby *reducing* the concentration of ozone-depleting radicals to some extent.

- It is not a major ozone-depleting substance.

🚫 *Not primarily responsible.*

🔷 Option D: Cl_2 (Chlorine gas)

- Though molecular chlorine can destroy ozone, it doesn't reach the stratosphere in large amounts due to high reactivity and solubility.
- However, under certain conditions, Cl_2 produced from CFC photolysis can release Cl atoms – so it is indirectly involved.

✅ *Can contribute, though not directly emitted into stratosphere.*

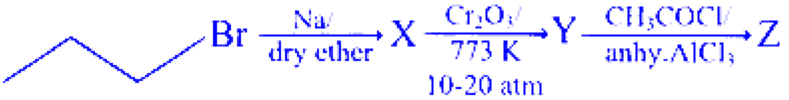
✅ **Final Answer:**

$\boxed{CH_4}$

Methane (CH_4) is **not primarily responsible** for depletion of the ozone layer in the stratosphere.

Question17

Consider the following sequence of reaction. In ' Z ' the number of sp^3 carbons is ' a ' and sp^2 carbons is ' b '. Value of $(a+b)$ is



Options:

A.

8

B.

7

C.

6

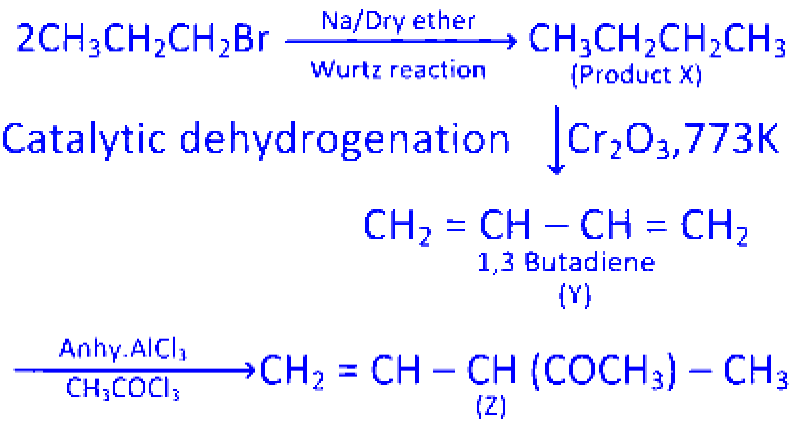
D.

9

Answer: A

Solution:

The complete reaction mechanism is as follows



So, in product Z, number of sp^2 carbon $(b) = 4$, number of sp^3 carbon $(a) = 4$

So, $a + b = 8$

Question18

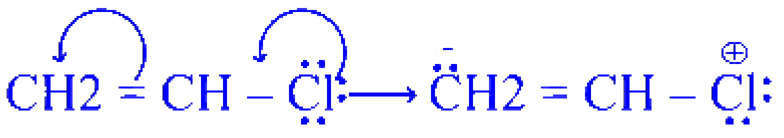
Which one of the following represents hyperconjugation effect?

Options:

A.

$$CH_3 - CH = CH_2 \longrightarrow \overset{H^{\ominus}}{CH_2} = CH - \ddot{C}H_2$$

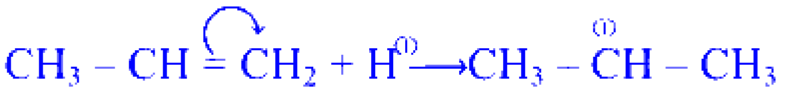
B.



C.

$$CH_3 - CH_2 \rightarrow NO_2 \longrightarrow CH_3 - \overset{\delta+}{CH_2} - \overset{\delta-}{NO_2}$$

D.



Answer: A

Solution:

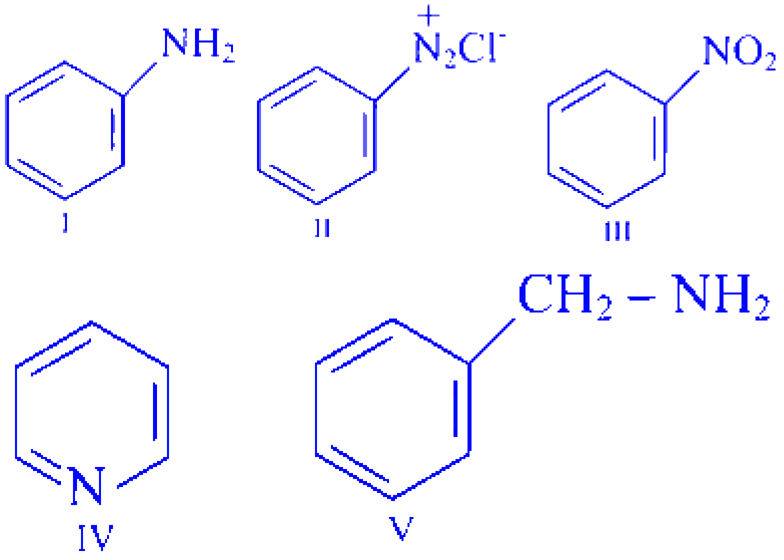
Hyperconjugation involves delocalisation of electrons from CH σ-bond adjacent to a π system or carbocation.

Hence, option (a) shows hyperconjugation.

$CH_3 - CH = CH_2$ molecules shows hyperconjugation as electron from CH bond of methyl group interact with π bond of alkene.

Question19

Which of the following compounds will be suitable for estimation of nitrogen by Kjeldahl's method?



Options:

A.

I and V only

B.

I, II, III only

C.

II and V only

D.

III and IV only

Answer: A

Solution:

Kjeldahl's method can estimate nitrogen only in organic compounds where nitrogen is present in amino or amide forms and not in aromatic rings, azo groups, or nitro groups.

Compound I (aniline) and compound(V) (benzylamine) is suitable.

Question20

For the alkyne with formula C_6H_{10}, the number of alkynes with acidic hydrogens is x and number of alkynes with no acidic hydrogens is y. x and y are respectively

Options:

A.

2,5

B.

3,4

C.

4,3

D.

5,2

Answer: C

Solution:

Given that molecular formula is C_6H_{10}. First of all let us calculate the degree of unsaturation for C_6H_{10}.

$$\begin{aligned} So,\ \mathrm{DBE} &= \mathrm{C} + 1\frac{-\mathrm{H}}{2} \\ \mathrm{DBE} &= 6 + 1\frac{-10}{2} = 2 \end{aligned}$$

Here, DBE of 2 indicates the presence of one triple bond or two double bonds or one ring and one double bond. So, for alkynes let us see the structure with one triple bond.

Now, the possible number of alkyne monomers of C_6H_{10} are

Hex-1-yne (Terminal alkyne)

Hex-2-yne (Internal alkyne)

Hex-3-yne (Internal alkyne)

3-methyl pent-1-yne (Terminal alkyne)

4-methylpent-1-yne (Terminal alkyne)

4-methylpent-2-yne (Internal alkyne)

3, 3-Dimethyl but-1-yne (Terminal alkyne)

So, terminal alkynes contain acidic hydrogen while internal alkynes contains no acidic hydrogen. Thus x and y are 4,3 respectively.

Question21

A substance has a density of $2 \mathrm{~g} \mathrm{~cm}^{-3}$. It crystallises in the fcc crystal with an edge length of 600 pm . The molar mass of the substance (in gmol^{-1}) is $\left(N_A=6 \times 10^{23} \mathrm{~mol}^{-1}\right)$

Options:

A.

54.8

B.

64.8

C.

74.8

D.

84.7

Answer: B

Solution:

Using the formula to calculate molar mass,

$M=\frac{d\times N_A\times a^3}{Z}$

For fcc crystal, $x=4$,

$$M=\frac{2\times 10^{23}\times 2.16\times 10^{-22}}{4}$$
$$=64.8\text{ g/mol}$$

Question22

Observe the following statements

Statement I : The boiling point of 0.1 M urea solution is less than that of 0.1 M KCl solution.

Statement II : Elevation of boiling point is inversely proportional to molar mass of solute.

The correct answer is

Options:

A.

Both Statements I and II are correct.

B.

Statement I is correct, but Statement II is not correct.

C.

Statement I is not correct, but Statement II is correct.

D.

Both Statement I and II are not correct.

Answer: A

Solution:

Both Statement I and II are correct.

Question23

At 298 K , if emf of the cell corresponding to the reaction $\mathrm{Zn}(s)+2\mathrm{H}^{+}(aq)\longrightarrow \mathrm{Zn}^{2+}(0.01\mathrm{M})+\mathrm{H}_2(g)(1\ \mathrm{atm})$ **is 0.28 V , then the pH of the solution at the hydrogen electrode is** $\left(\frac{2.303RT}{F}=0.06\ \mathrm{V}\right),\left(E^{\circ}_{\mathrm{Zn}^{2+}/\mathrm{Zn}}=-0.76\ \mathrm{V}\right)$

Options:

A.

8

B.

7

C.

9

D.

10

Answer: C

Solution:

The cell reaction is,

At anode (Zn), $\mathrm{Zn}\longrightarrow \mathrm{Zn}^{2+}+2e^{-}$

Cathode (H^{+}), $2\mathrm{H}^{+}+2e^{-}\longrightarrow \mathrm{H}_2$

Using Nernst equation,

$$E_{\text{cell}}=E^{\circ}_{\text{cathode}}-E^{\circ}_{\text{anode}}-\frac{2.303RT}{F}\log Q$$

$$0.28 = +0.76 - \frac{0.06}{2} \log \left(\frac{0.01}{[H^+]^2} \right)$$

Solving this,

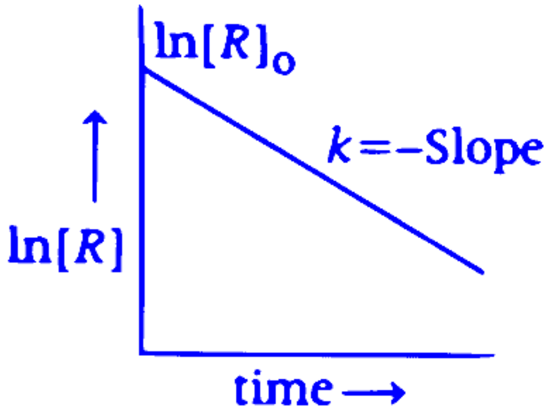
$$[H^+] = 10^{-9}$$
$$pH = -\log [H^+] = 9$$

Question24

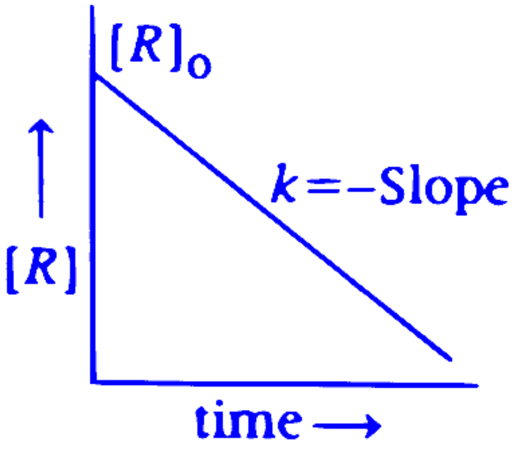
For the reaction $R \rightarrow P$, half life is independent of initial concentration of the reactant, R. Which one of the following graphs is not correct for the reaction?

Options:

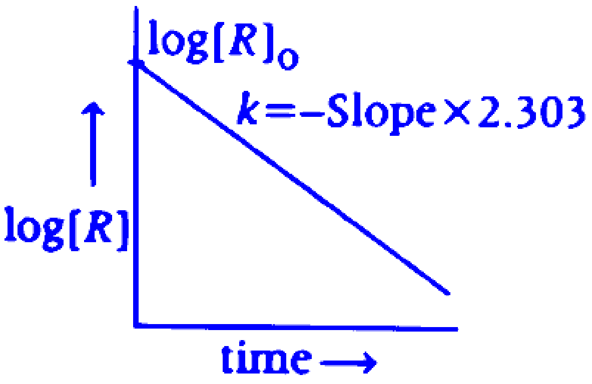
A.



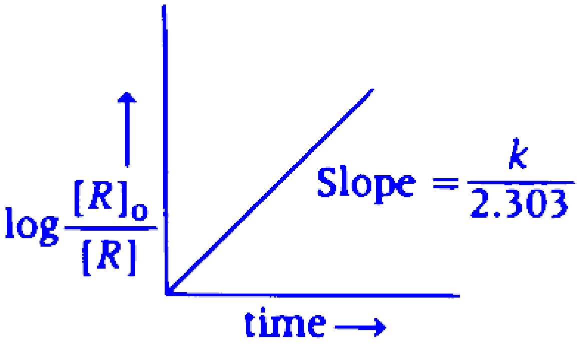
B.



C.



D.



Answer: B

Solution:

$R \longrightarrow P$

Half-life is independent of initial concentration, means it's a first order reaction.

Integrated rate law,

$\ln[R] = -kt + \ln\left[R_0\right]$.

Thus, $\ln R vs$ time is a straight line with slope $= -k$.

Option (b) graph is incorrect because it $[R]$ vs t.

So, it should be exponential.

Question25

Which of the following is not correct about Freundlich adsorption isotherm?

Options:

A.

$\frac{x}{m} = k_p^{1/n}(n > 1)$

B.

Extent of adsorption of gas is more at high temperature than at low temperature

C.

$\frac{1}{n}$ represents the slope of the isotherm

D.

$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$ holds good over a limited range of pressures

Answer: B

Solution:

Freundlich adsorption isotherm equation:

$\frac{x}{m} = k p^{1/n}$

where

- $\frac{x}{m}$ = mass of gas adsorbed per unit mass of adsorbent,
- p = equilibrium pressure,
- k and n = empirical constants,
- $n > 1$.

Or, taking logarithms,

$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$

Option A:

$\frac{x}{m} = k p^{1/n} \quad (n > 1)$

✅ **Correct.**

Option B:

Extent of adsorption of gas is more at high temperature than at low temperature.

Adsorption is an **exothermic process**.

Hence, increasing temperature **decreases** adsorption (Le Chatelier's principle).

❌ **Incorrect statement.**

Option C:

$\frac{1}{n}$ represents the slope of the isotherm (in log–log plot)

Since $\log \frac{x}{m} = \log k + \frac{1}{n} \log p$,

✅ **Correct.**

Option D:

$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$ holds good over a limited range of pressures.

At very high or very low pressures, Freundlich isotherm deviates.

✅ **Correct.**

✅ **Final Answer:**

Option B — *Extent of adsorption of gas is more at high temperature than at low temperature* — is **not correct** about Freundlich adsorption isotherm.

Question26

Identify the reaction, which is not related to extraction of copper

Options:

A.

$2Cu_2 S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$

B.

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$

C.

$FeO + SiO_2 \longrightarrow FeSiO_3$

D.

$SiO_2 + CaO \longrightarrow CaSiO_3$

Answer: D

Solution:

Reaction (a) i.e

$2Cu_2\,S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$

copper extraction via roasting.

Reaction (b) i.e

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$ is use to remove iron oxide from copper ore.

Reaction (c) i.e

$FeO + SiO_2 \longrightarrow FeSiO_3$ is removal of slag in copper extraction.

$SiO_2 + CaO \longrightarrow CaSiO_3$ is not related to copper extraction.

Question27

Phosphorus on reaction with sulphuryl chloride gives a compound X, which on complete hydrolysis gives Y. X and Y are respectively.

Options:

A.

PCl_3, H_3PO_3

B.

$PCl_5, POCl_3$

C.

PCl_5, H_3PO_4

D.

PCl_3, H_3PO_2

Answer: C

Solution:

The complete reaction is as follows,

$$P + SO_2Cl_2 \longrightarrow \underset{\text{(Product X)}}{PCl_5}$$

$$PCl_5 + H_2O \longrightarrow \underset{\text{(Product Y)}}{H_3PO_4}$$

Complete hydrolysis, Thus, $X = PCl_5, Y = H_3PO_4$.

Question28

Xenon hexafluoride on partial hydrolysis gives ' X ' and HF . The shape of ' X ' is

Options:

A.

pyramidal

B.

tetrahedral

C.

square pyramidal

D.

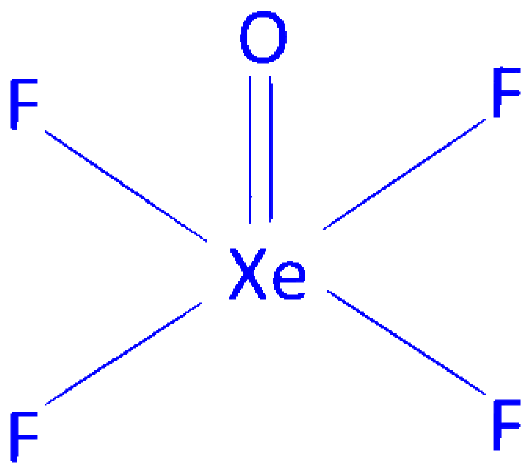
linear

Answer: C

Solution:

The reaction involved is,

$$XeF_6 + H_2O \longrightarrow \underset{(\text{Product} \quad X)}{XeOF_4} + 2HF$$



Thus the shape of $(X) XeF_4$ is square pyramidal.

Question29

Which of the following pairs of oxoacids have basicity as 2?

Options:

A.

H_3PO_3, H_2SO_4

B.

H_3PO_2, H_2SO_3

C.

H_3PO_4, H_3PO_2

D.

$H_2\ S_2O_8, H_3PO_2$

Answer: A

Solution:

H_3PO_3 and H_2SO_4 both have basicity of 2. H_3PO_3 has two $P-OH$ bond meaning it can donate two $[OH^-]$ ions when in solution.

H_2SO_4 also have two ionisation H -atom attached to oxygen, giving it a basicity of 2 .

Question30

In acidic medium one mole each of MnO_4^- and $Cr_2O_7^{2-}$ is reduce by x and y moles of ferrous ions. The sum of x and y is

Options:

A.

14

B.

12

C.

10

D.

11

Answer: D

Solution:

In acidic medium,

$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$

1 mole of MnO_4^- requires 5 moles of $Fe^{2+} x = 5$

$Cr_2O_7^{2-} + 14H^+ + 6Fe^{2+} \longrightarrow 2Cr^{3+} + 7H_2O + 6Fe^{3+}$

1 mole of $Cr_2O_7^{2-}$ are needed to give 6 moles of Fe^{3+}, $y = 6$.

$x + y = 5 + 6 = 11$

Question31

Which one of the following is not an ambidenate ligand?

Options:

A.

CN

B.

SCN^-

C.

SO_4^{2-}

D.

NO_2^-

Answer: C

Solution:

An ambidentate ligand is a ligand that can coordinate to a central metal atom through two different donor atoms, but not at the same time. SO_4^{2-} is a bidentate ligand.

Question32

' X ' is a polymer, which is mainly used for making unbreakable cups and laminated sheets. The monomers of ' X ' are

Options:

A.

urea and formaldehyde

B.

ethylene glycol and phthalic acid

C.

phenol and formaldehyde

D.

1,3-butadiene and styrene

Answer: C

Solution:

Given:

Polymer **'X'** is mainly used for making **unbreakable cups** and **laminated sheets**.

We're asked: What are the **monomers** of polymer X?

Step 1: Identify the polymer

- **Unbreakable cups** and **laminated sheets** are typically made from **melamine-formaldehyde** or **phenol-formaldehyde** type resins.
- However, **phenol-formaldehyde resin** (commonly known as **Bakelite**) is specifically known for:
- Being **hard and brittle**
- Used in **electrical switches**, **unbreakable crockery**, **laminated sheets**, etc.

Step 2: Match with monomers

Bakelite (phenol-formaldehyde resin) is formed by:

$\text{Phenol} + \text{Formaldehyde} \longrightarrow \text{Phenol-formaldehyde resin (Bakelite)}$

Hence, the monomers are **phenol** and **formaldehyde**.

✅ Correct Answer:

Option C: phenol and formaldehyde

Question33

Which of the following hormones is an example of polypeptide?

Options:

A.

Epinephrine

B.

Insulin

C.

Estrogen

D.

Androgen

Answer: B

Solution:

The correct answer is:

✅ **Option B: Insulin**

Explanation:

- **Polypeptide hormones** are made up of chains of amino acids.
- **Insulin** is a classic example — it's composed of two peptide chains (A and B chains) linked by disulfide bonds.
- **Epinephrine** is a **catecholamine** (derived from the amino acid tyrosine).
- **Estrogen** and **androgen** are **steroid hormones** derived from cholesterol.

Hence, **Insulin** is the **polypeptide hormone** among the options.

Question34

The structure of which artificial sweetener contains aspartic acid and phenylalanine parts?

Options:

A.

Saccharin

B.

Sucralose

C.

Alitame

D.

Aspartame

Answer: D

Solution:

The correct answer is:

✅ **Option D: Aspartame**

Explanation:

Aspartame is a dipeptide methyl ester composed of **aspartic acid** and **phenylalanine**. Its chemical name is *L-aspartyl-L-phenylalanine methyl ester*.

Because it contains phenylalanine, individuals with **phenylketonuria (PKU)** must avoid it.

Other options for comparison:

- **Saccharin** – a sulfonamide compound, not derived from amino acids.
- **Sucralose** – a chlorinated derivative of sucrose (sugar), not protein-based.
- **Alitame** – a dipeptide sweetener related to aspartame but has a different amine component (aspartic acid + a different amine, not phenylalanine).

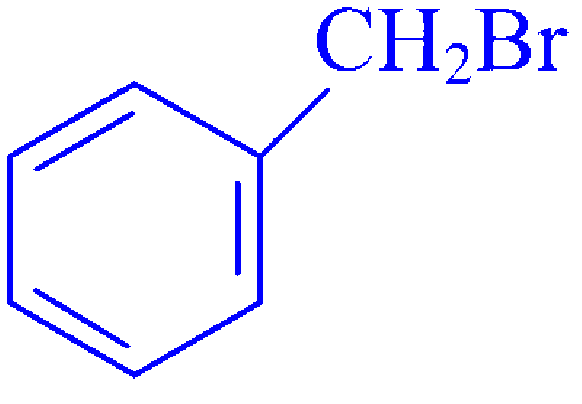
Answer: D) Aspartame

Question35

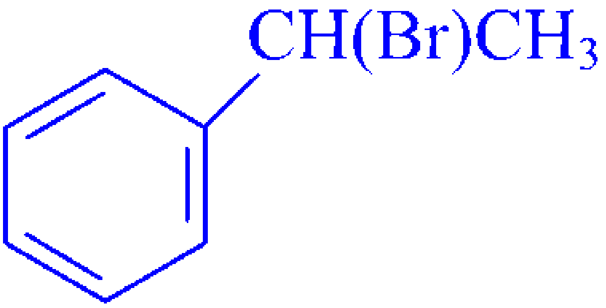
Which of the following is the most reactive towards S_N1 mechanism?

Options:

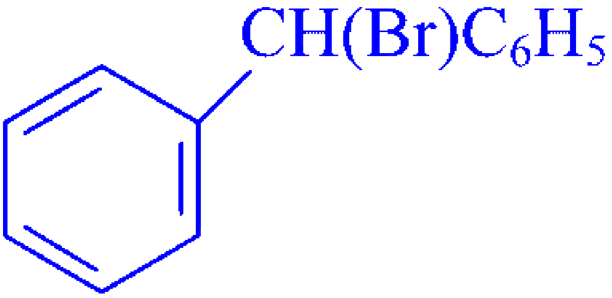
A.



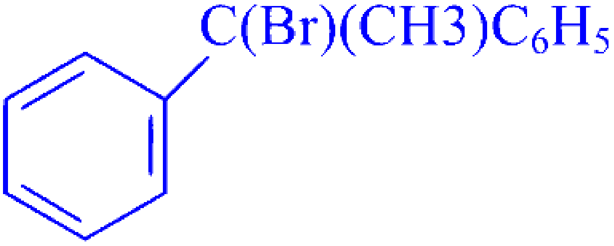
B.



C.



D.

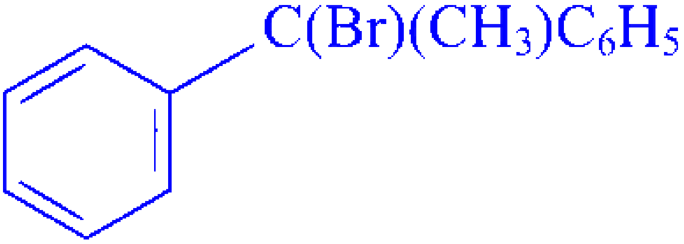


Answer: D

Solution:

S_N1 mechanism depends on carbocation stability. The more stable the carbocation formed after the leaving group departs, the faster is the S_N1 reaction.

In



tertiary benzylic carbocation is stabilised by $+I$ effect of CH_3 and 2 phenyl group, which is most stable among the given options.

Question36

$$(CH_3)_3CH \xrightarrow{KMnO_4} X \xrightarrow[573\ K]{Cu} Y$$

The number of sp^3 and sp^2 carbons in Y are respectively

Options:

A.

3,1

B.

2,2

C.

1,3

D.

4,0

Answer: C

Solution:

The complete reaction is as follows,

$$(CH_3)_3CH \xrightarrow{KMnO_4} \underset{\text{Product } X}{(CH_3)_3COH} \xrightarrow[573\ K]{Cu} \underset{\substack{\text{(Product Y)} \\ \text{Isobutene}}}{(CH_3)_2C = CH_2}$$

Number of sp^3 carbon $= 2$ (methyl group)

Number of sp^2 carbon $= 2$(double bond carbon)

Question37

Consider the following reaction sequence. (A) and (C) are

$$CH_3CHO \xrightarrow[\text{(ii)}\quad H_2O/H^+]{\text{(i)}\quad CH_3MgBr} (A) \xrightarrow{H_5SO_4.\Delta} (B) \xrightarrow[\text{(ii)}\quad H_2O,H_2O_2/OH^-]{\text{(i)}\quad B_3H_6} (C)$$

Options:

A.

functional isomers

B.

metamers

C.

optical isomers

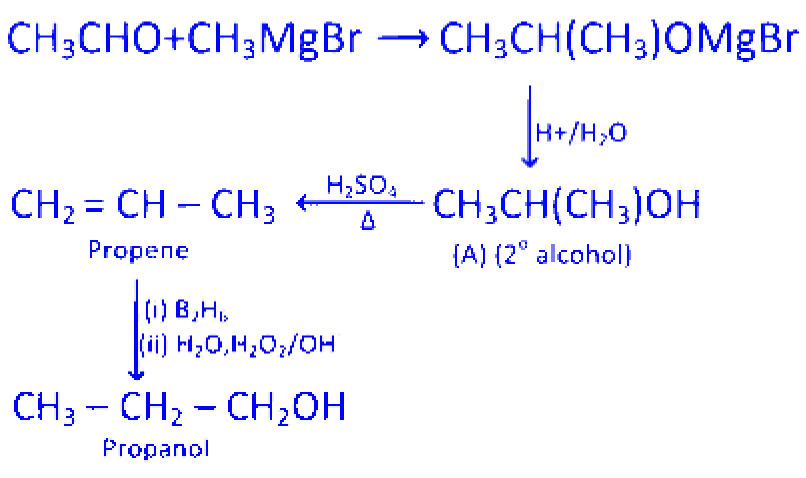
D.

position isomers

Answer: D

Solution:

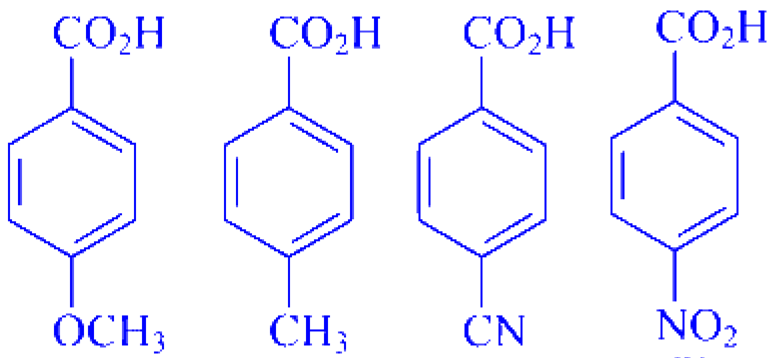
The complete reaction is as follows,



Hence, A and C are position isomers.

Question38

The increasing order of acidic strength of the following in aqueous solution is



I II III IV

Options:

A.

IV < II < III < I

B.

I < III < II < IV

C.

I < II < III < IV

D.

III < I < II < IV

Answer: C

Solution:

The correct increasing order of acidic strength of given compounds is I < II < III < IV.

As acidity increases with electron withdrawing group.

$-OCH_3 < -CH_3 < -CN < -NO_2$

Question39

The increasing order of boiling points of the following is

$$\underset{\text{I}}{CH_3 - O - CH_3} \quad \underset{\text{II}}{CH_3CHO} \quad \underset{\text{III}}{CH_3CH_2CH_3} \quad \underset{\text{IV}}{CH_3CH_2OH}$$

Options:

A.

I < III < II < IV

B.

III < I < II < IV

C.

I < IV < III < II

D.

III < I < IV < II

Answer: B

Solution:

The correct increasing order of boiling points is

$$\underset{\text{(III)}}{CH_3CH_2CH_3} < \underset{\text{(I)}}{CH_3 - O - CH_3} < \underset{\text{(II)}}{CH_3CHO} < \underset{\text{(IV)}}{CH_3CH_2OH}$$

It is because of strength of intermolecular forces present in each molecule, with hydrogen bonding in alcohols being the strongest and van der Waals' forces in alkanes being the weakest.

Question40

The major products P and Q from the following reactions are

$$P \xleftarrow[H_2O]{LiAlH_4} C_6H_5CONH_2 \xrightarrow{Br_2/NaOH} Q$$

Options:

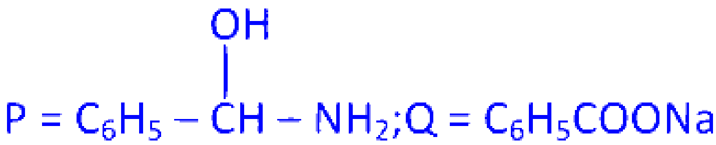
A.

$P = C_6H_5NH_2; Q = C_6H_5CH_2NH_2$

B.

$P = C_6H_5CH_2NH_2; Q = C_6H_5NH_2$

C.



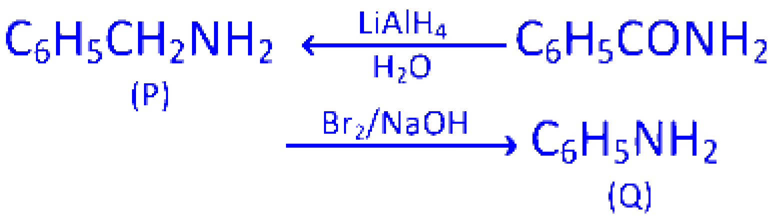
D.

$P = C_6H_5CN; Q = C_6H_5Br$

Answer: B

Solution:

The complete reaction is as follows.



Physics

Question1

For any fixed distance, the electromagnetic force between two protons is 10^n times of the gravitational force between them. Then, $n =$

Options:

A.

26

B.

13

C.

39

D.

36

Answer: D

Solution:

The electromagnetic force between two protons is much stronger than the gravitational force between them.

Let F_e be the electromagnetic force, and F_g be the gravitational force.

According to the problem: $F_e = 10^n F_g$

We write the formulas for both forces:

Electromagnetic force: $F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

Gravitational force: $F_g = \frac{Gm^2}{r^2}$

Replace F_e and F_g in the equation: $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = 10^n \frac{Gm^2}{r^2}$

Since r^2 is on both sides, it cancels out:

$\frac{1}{4\pi\varepsilon_0}e^2 = 10^n Gm^2$

Solve for 10^n:

$10^n = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{Gm^2}$

Now, use the constants:

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

Charge of proton, $e = 1.6 \times 10^{-19}$

Gravitational constant, $G = 6.67 \times 10^{-11}$

Mass of proton, $m = 1.67 \times 10^{-27}$

Plug in the values:

$10^n = \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times(1.67\times10^{-27})^2}$

Calculate the numerator: $9 \times 10^9 \times (1.6 \times 10^{-19})^2 = 9 \times 10^9 \times 2.56 \times 10^{-38} = 2.304 \times 10^{-28}$

Calculate the denominator:

$6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2 = 6.67 \times 10^{-11} \times 2.79 \times 10^{-54} = 1.86 \times 10^{-64}$

Divide the values: $\frac{2.304 \times 10^{-28}}{1.86 \times 10^{-64}} = 1.23 \times 10^{36}$

This means $10^n = 1.23 \times 10^{36}$, so $n = 36$.

Question2

If A, B and C are three different physical quantities with different dimensional formulae, then the combination which can never give a proper physical quantity is

Options:

A.

$\frac{A}{BC}$

B.

$\frac{AB-C^2}{BC}$

C.

$\frac{A-C}{B}$

D.

$AC - B$

Answer: C

Solution:

According to principle of homogenity, physical quality can only be added or subtracted if they have the same dimensions.

$\frac{A-C}{B}$ is correct only when A and C must have some dimensions.

Question3

The driver of a bus moving with a velocity of $72 \mathrm{~km} / \mathrm{h}$ observes a boy walking across the road at a distance of 50 m in front of the bus and decelerates the bus at $5 \mathrm{~ms}^{-2}$ by applying brakes and is just able to avoid an accident. The reaction time of the driver is

Options:

A.

4 s

B.

3.5 s

C.

0.5 s

D.

4.5 s

Answer: C

Solution:

$u=72 \mathrm{~km} / \mathrm{h}=72 \times \frac{3}{18}=20 \mathrm{~m} / \mathrm{s}$

$a=-5 \mathrm{~m} / \mathrm{s}^2$

$\therefore$ using , $v^2=u^2+2 a s$

$\Rightarrow 0=20^2+2(-5) s$

$\Rightarrow s=\frac{400}{10}=40 \mathrm{~m}$

Distance travelled during reaction time

$=$ Total distance $-s$

$=50-40=10 \mathrm{~m}$

If t be the reaction time, then

$$s = ut$$
$$t = \frac{s}{u} = \frac{10}{20} = 0.5 \text{ s}$$

Question4

A helicopter flying horizontally with a velocity of 288 km/h drops a bomb. If the line joining the point of dropping the bomb and the point where bomb hits the ground makes an angle 45° with the horizontal, then the height at which the bomb was dropped is (Acceleration due to gravity $= 10 \text{ ms}^{-2}$)

Options:

A.

1320 m

B.

1280 m

C.

320 m

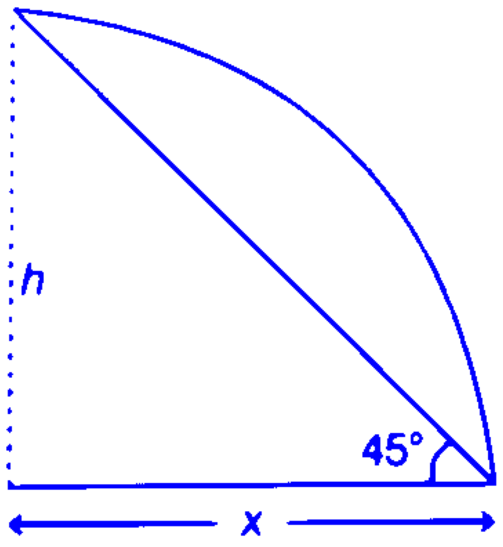
D.

640 m

Answer: B

Solution:

$$u = 288 \text{ km/h}$$
$$= 288 \times \frac{5}{18} = 80 \text{ m/s}$$
$$h = \frac{1}{2} g t^2$$
$$x = ut$$



$\therefore \quad \tan 45^\circ = \frac{h}{x}$

$\Rightarrow \quad h = x \Rightarrow \frac{1}{2}gt^2 = ut$

$\Rightarrow \quad t = \frac{2u}{g} = \frac{2\times 80}{10} \Rightarrow t = 16 \text{ s}$

$\therefore$ Height of bomb, $h = \dfrac{1}{2}gt^2$

$= \dfrac{1}{2} \times 10 \times 16^2 = 1280 \text{ m}$

Question5

A man of mass 60 kg is standing in a lift moving up with a retardation of 2.8 ms^{-2}. The apparent weight of the man is

Options:

A.

756 N

B.

168 N

C.

588 N

D.

420 N

Answer: D

Solution:

When a man standing in a lift moving up with a retardation of $a\ \mathrm{m/s^2}$, then apparent weight $= m(g+a)$

$$= 60(9.8-2.8) \therefore a = -2.8\ \mathrm{m/s^2}$$
$$= 60 \times 7 = 420\ \mathrm{N}$$

Question6

The initial and final velocities of a body projected vertically from the ground are $20\ \mathrm{ms}^{-1}$ and $18\ \mathrm{ms}^{-1}$ respectively. The maximum height reached by the body is (Acceleration due to gravity $= 10\ \mathrm{ms}^{-2}$)

Options:

A.

20 m

B.

16.2 m

C.

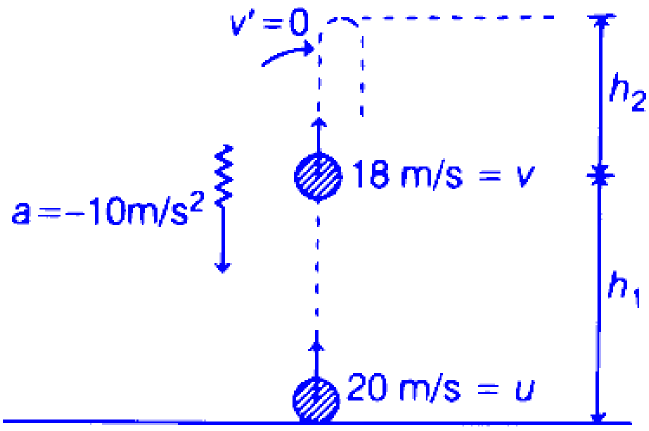
19 m

D.

18.1 m

Answer: A

Solution:



Using $v^2 - u^2 = 2gh$

We have for h_1 (see figure)

$$18^2 - 20^2 = 2(-10) \times h_1$$
$$\Rightarrow \quad h_1 = 3.8 \mathrm{~m}$$

Now for h_2,

$$0 - 18^2 = 2(-10) \times h_2$$
$$\Rightarrow \quad h_2 = 16.2 \mathrm{~m}$$

Maximum height attained

$$= h_1 + h_2 = 3.8 + 16.2$$
$$= 20 \mathrm{~m}$$

Question7

A particle is acted upon by a force of constant magnitude such that its velocity and acceleration are always perpendicular to each other, then its

Options:

A.

linear momentum is constant

B.

kinetic energy is constant

C.

velocity is constant

D.

acceleration is constant

Answer: B

Solution:

We know that force ($\mathbf{F}$) is directly proportional to acceleration (a) by Newton's second law ($\mathbf{F}=m\mathbf{a}$). Therefore, if the force is always perpendicular to the velocity, then the acceleration is also always perpendicular to the velocity. The work done by a force is given by $W=\int \mathbf{F} \cdot d\mathbf{s}$, where $d\mathbf{s}$ is the displacement.

Since, $\mathbf{F}$ is perpendicular to $\mathbf{v}$ and $d\mathbf{s}$ is in the direction of $\mathbf{v}$, the dot product $\mathbf{F} \cdot d\mathbf{s}=0$. This means the work done by the force is zero.

According to the work-energy theorem, the net work done on a particle equals the change in its kinetic energy ($\Delta \mathrm{KE}$). Since the work done is zero, the change in kinetic energy is zero, meaning the kinetic energy remains constant.

Question8

If the moment of inertia of a uniform solid cylinder about the axis of the cylinder is $\frac{1}{n}$ times its moment of inertia about an axis passing through its midpoint and perpendicular to its length, then the ratio of the length and radius of the cylinder is

Options:

A.

$\sqrt{2(3n+1)}$

B.

$\sqrt{2(3n-1)}$

C.

$\sqrt{3(2n+1)}$

D.

$\sqrt{3(2n-1)}$

Answer: D

Solution:

As we know,

Moment of inertia about axis of cylinder,

$I_{\text{long}} \quad = \frac{1}{2} MR^2$

Moment of inertia about an axis through centre and perpendicular to length.

$I_{\text{transverse}} \quad = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$

By using, given ratio,

$$\frac{1}{2} MR^2 = \frac{1}{n}\left(\frac{1}{12} ML^2 + \frac{1}{4} MR^2\right)$$
$$\left(\frac{2n-1}{4}\right) R^2 = \frac{1}{12} L^2$$
$$3(2n-1) = \frac{L^2}{R^2} \Rightarrow \frac{L}{R} = \sqrt{3(2n-1)}$$

Question9

Two blocks of masses in the ratio $m : n$ are connected by a light inextensible string passing over a frictionless fixed pulley. If the system of the blocks is released from rest, then the acceleration of the centre of mass of the system of the blocks is

($g =$ acceleration due to gravity)

Options:

A.

$\left(\frac{m+n}{m-n}\right)^2 g$

B.

$\left(\frac{m-n}{m+n}\right)^2 g$

C.

$\left(\frac{m+n}{m-n}\right) g$

D.

$\left(\frac{m-n}{m+n}\right)g$

Answer: B

Solution:

For block m (moving down)

$mg - T = ma \qquad \ldots(i)$

For block n (moving up)

$T - ng = na \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$mg - ng = ma + na$$
$$(m-n)g = (m+n)a$$
$$a = \frac{(m-n)}{(m+n)}g$$

$\therefore \quad a_m = +a$ and $a_n = -a$

then, $a_{CM} = \frac{ma-na}{m+n} = \frac{(m-n)}{(m+n)} \times a$

$a_{CM} = \left(\frac{m-n}{m+n}\right)^2 g$

Question10

The amplitude of a particle executing simple harmonic motion is 6 cm . The distance of the point from the mean position at which the ratio of the potential and kinetic energies of the particle becomes 4 : 5 is

Options:

A.

6 cm

B.

4 cm

C.

3 cm

D.

2 cm

Answer: B

Solution:

Given:

$\frac{U}{K} = \frac{4}{5}$

The amplitude $A = 6$ cm

Step 1: Write the energy formulas

The total energy is $E = \frac{1}{2}kA^2$

The potential energy is $U = \frac{1}{2}kx^2$

The kinetic energy is $K = \frac{1}{2}k(A^2 - x^2)$

Step 2: Set up the given ratio

We want the distance x from the mean position so that $\frac{U}{K} = \frac{4}{5}$:

$\frac{\frac{1}{2}kx^2}{\frac{1}{2}k(A^2-x^2)} = \frac{4}{5}$

Step 3: Simplify the equation

The $\frac{1}{2}k$ cancels out: $\frac{x^2}{A^2-x^2} = \frac{4}{5}$

Step 4: Solve for x^2

Multiply both sides by $(A^2 - x^2)$: $5x^2 = 4(A^2 - x^2)$

Expand: $5x^2 = 4A^2 - 4x^2$

Bring $4x^2$ to the left: $5x^2 + 4x^2 = 4A^2$

Combine like terms: $9x^2 = 4A^2$

Step 5: Find x

Divide both sides by 9: $x^2 = \frac{4}{9}A^2$

Take the square root: $x = \frac{2}{3}A$

Since $A = 6$ cm:

$x = \frac{2}{3} \times 6 = 4$ cm

The distance from the mean position is 4 cm.

Question11

If a body is projected vertically from the surface of the Earth with a speed of $8000 \mathrm{~ms}^{-1}$, then the maximum height reached by the body is

(Radius of the Earth $= 6400 \mathrm{~km}$ and acceleration due to gravity $= 10 \mathrm{~ms}^{-2}$)

Options:

A.

1600 km

B.

9600 km

C.

6400 km

D.

3200 km

Answer: C

Solution:

Applying conservation of energy

$\frac{1}{2} m v^2-\frac{G M m}{R}=-\frac{G M m}{R+h}$

But $G M=g R^2$

$\therefore \quad \frac{1}{2} m v^2-\frac{g R^2 m}{R}=\frac{-g R^2 m}{R+h}$

$$\Rightarrow \quad \frac{v^2}{2} - gR = -\frac{gR^2}{R+h}$$
$$\Rightarrow \quad h = R\left(\frac{v^2}{2gR - v^2}\right)$$
$$= 6.4 \times 10^6 \left[\frac{(8000)^2}{2 \times 10 \times 6.4 \times 10^6 - (8000)^2}\right]$$
$$= 6.4 \times 10^6 \text{ m} = 6400 \times 10^3 \text{ m}$$
$$= 6400 \text{ km}$$

Question12

If a brass sphere of radius 36 cm is submerged in a lake at a depth where the pressure is 10^7 Pa, then the change in the radius of the sphere is

(Bulk modulus of brass $= 60 \mathrm{GPa}$)

Options:

A.

4×10^{-2} cm

B.

2×10^{-3} cm

C.

4×10^{-3} cm

D.

2×10^{-2} cm

Answer: B

Solution:

$$B = \frac{F}{A} \times \frac{V}{\Delta V}$$
$$B = \frac{PV}{\Delta V}$$

$$Here,\ V=\frac{4}{3}\pi R^3$$
$$\Delta V=4\pi R^2\Delta R$$

$$\therefore \quad B=P\times\frac{4}{3}\pi R^3\times\frac{1}{4\pi R^2\Delta R}$$
$$B=\frac{PR}{3\Delta R}$$
$$\Delta R=\frac{PR}{3B}=\frac{10^7\times 36\times 10^{-2}}{3\times 60\times 10^9}$$
$$\Delta R=2\times 10^{-5}\ \mathrm{m}$$
$$=2\times 10^{-3}\ \mathrm{cm}$$

Question13

The work done in blowing a soap bubble of diameter 3 cm is (surface tension of soap solution $=0.035\mathrm{Nm}^{-1}$)

Options:

A.

792μ J

B.

99μ J

C.

396μ J

D.

198μ J

Answer: D

Solution:

$d=3$ cm
$r=\frac{3}{2}\times 10^{-2}$ m
$T=0.035\mathrm{Nm}^{-1}$

For soap bubble, both inner and outer surface contributes, so increase in surface area is

$\Delta A=2\times 4\pi r^2=8\pi r^2$

Work done = Surface tension × Increase in area

$$W = T \times \Delta A$$
$$= 0.035 \times 8 \times \pi \times \left(\frac{3}{2} \times 10^{-2}\right)^2$$
$$\approx 1.98 \times 10^{-4}\text{ J} \approx 198\mu\text{ J}$$

Question14

If the terminal velocity of a metal sphere of mass 8 g falling through a liquid is 3cms^{-1}, then the terminal velocity of another sphere of mass 64 g made of the same metal falling through same liquid is

Options:

A.

6cms^{-1}

B.

3cms^{-1}

C.

12cms^{-1}

D.

18cms^{-1}

Answer: C

Solution:

We know that terminal velocity

$$v = \frac{2r^2(\rho-\sigma)g}{9\eta}$$
$$\Rightarrow \quad v \propto r^2 \qquad \ldots(i)$$

But mass of sphere

$$m = \frac{4}{3}\pi r^3 \rho$$
$$\Rightarrow \quad r^3 \propto m$$
$$\Rightarrow \quad r \propto m^{1/3} \qquad \ldots(i)$$

From Eqs. (i) and (ii), we have

$$v \propto \left(m^{1/3}\right)^2$$

$$\Rightarrow \quad v \propto m^{2/3}$$

$$\Rightarrow \quad \frac{v_2}{v_1} = \left(\frac{m_2}{m_1}\right)^{2/3}$$

$$= \left(\frac{64}{8}\right)^{2/3} = (8)^{2/3}$$

$$= \left(2^3\right)^{2/3} = 2^2 = 4$$

$$\Rightarrow \quad v_2 = 4v_1 = 4 \times 3 = 12 \text{ cm/s}$$

Question15

The length of a metal rod is 20 cm and its area of cross-section is 4 cm^2. If one end of the rod is kept at a temperature of $100^{\circ}\mathrm{C}$ and the other end is kept in ice at $0^{\circ}\mathrm{C}$, then the mass of the ice melted in 7 minutes is (Thermal conductivity of the metal $= 90 \mathrm{Wm}^{-1} \mathrm{~K}^{-1}$ and latent heat of fusion of ice $= 336 \times 10^3 \mathrm{Jkg}^{-1}$)

Options:

A.

90 g

B.

67.5 g

C.

22.5 g

D.

45 g

Answer: C

Solution:

The heat transferred through the rod is

$Q = \frac{kA\Delta T}{l}t$

$$Q = \frac{90\ \mathrm{W/mk} \times 4 \times 10^{-4}\ \mathrm{m}^2 \times (100^\circ\mathrm{C} - 0^\circ\mathrm{C})}{0.2\ \mathrm{m}} \times 420\ \mathrm{s}$$

$$Q = \frac{90 \times 4 \times 10^{-4} \times 100}{0.2} \times 420\ \mathrm{J}$$

$$Q = \frac{36000 \times 10^{-4}}{0.2} \times 420\ \mathrm{J}$$

$$Q = \frac{3.6}{0.2} \times 420\ \mathrm{J}$$

$$Q = 18 \times 420\ \mathrm{J} = 7560\ \mathrm{J}$$

The heat required to melt a mass m of ice is given by $Q = mL_f$

$$m = \frac{Q}{L_f}$$

$$m = \frac{7560\ \mathrm{J}}{336 \times 10^3\ \mathrm{J/kg}}$$

$$m = \frac{7560}{336000}\ \mathrm{kg}$$

$$m = 0.0225\ \mathrm{kg}$$

$$m = 22.5\ \mathrm{g}$$

The mass of ice melted is 22.5 g

Question16

The heat required to convert 8 g of ice at a temperature of $-20^\circ\mathrm{C}$ to steam at $100^\circ\mathrm{C}$ is [specific heat capacity of ice $= 2100\mathrm{Jkg}^{-1}\ \mathrm{K}^{-1}$, specific heat capacity of water $= 4200\ \mathrm{J\ kg}^{-1}\ \mathrm{K}^{-1}$, latent heat of fusion of ice $= 336 \times 10^3\ \mathrm{J\ kg}^{-1}$ and latent heat of steam $= 2.268 \times 10^6 \mathrm{Jkg}^{-1}$]

Options:

A.

5400 cal

B.

5840 cal

C.

5760 cal

D.

5120 cal

Answer: B

Solution:

Heat of raise temperature of ice from $-20^{\circ}\mathrm{C}$ to $0^{\circ}\mathrm{C}$.

$$\begin{aligned} Q_1 &= m \cdot C_{\text{ice}} \cdot \Delta T \\ &= 0.008 \times 2100 \times 20 = 336 \mathrm{~J} \end{aligned}$$

Heat to melt ice at $0^{\circ}\mathrm{C}$ to water

$$Q_2 = m \times L_f = 0.008 \times 336000 = 2,688 \mathrm{~J}$$

Heat to raise temperature of water from $0^{\circ}\mathrm{C}$ to $100^{\circ}\mathrm{C}$

$$\begin{aligned} Q_3 &= m \times C_{\text{water}} \times \Delta T \\ &= 0.008 \times 4200 \times 100 = 3360 \mathrm{~J} \end{aligned}$$

Heat to convert water at $100^{\circ}\mathrm{C}$ to steam

$$\begin{aligned} Q_4 &= M \cdot L_v = 0.008 \times 2.268 \times 10^6 \\ &= 18144 \mathrm{~J} \end{aligned}$$

Total heat required

$$\begin{aligned} Q_{\text{Total}} &= Q_1 + Q_2 + Q_3 + Q_4 \\ &= 336 + 2688 + 3360 + 18144 \\ &= 24,528 \mathrm{~J} (\text{ or } 24.5 \mathrm{~kJ}) \end{aligned}$$

1 calorie $= 4.18 \mathrm{~J}$

Total heat in joules

$$Q = \frac{24528}{4.2} \approx 5840 \mathrm{cal}$$

Question17

Two moles of a gas at a temperature of $327^{\circ}\mathrm{C}$ expands adiabatically such that its volume increases by 700%. If the ratio of the specific heat capacities of the gas is $\frac{4}{3}$, then the work done by the gas is (Universal gas constant $= 8.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$)

Options:

A.

14.94 kJ

B.

29.88 kJ

C.

44.82 kJ

D.

59.76 kJ

Answer: A

Solution:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$
$$T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1} = 600\left[\frac{V}{8V}\right]^{\frac{4}{3}-1}$$

$$T_2 = 600 \times \frac{1}{2} = 300 \text{ K}$$
$$W = \frac{nRT_1}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]$$
$$= \frac{2 \times 8.3 \times 600}{\frac{4}{3} - 1}\left[1 - \left(\frac{1}{8}\right)^{\frac{4}{3}-1}\right]$$
$$= \frac{3 \times 2 \times 8.3 \times 600}{1}\left[1 - \frac{1}{2}\right]$$
$$= \frac{3 \times 2 \times 8.3 \times 600}{2} = 14940 \text{ J}$$
$$= 14.94 \text{ kJ}$$

Question18

The molar specific heat of a monoatomic gas at constant pressure is

(Universal gas constant $= 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$)

Options:

A.

$24.9 \text{ J mol}^{-1} \text{ K}^{-1}$

B.

$20.75 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$

C.

$41.5 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$

D.

$16.6 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$

Answer: B

Solution:

Step 1: Recall relations

For an ideal gas,

$C_p - C_v = R$

For a **monoatomic** gas,

$C_v = \frac{3}{2}R$

So,

$C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$

Step 2: Substitute the gas constant

Given:

$R = 8.3 \, \mathrm{J\,mol^{-1}\,K^{-1}}$

Hence,

$C_p = \frac{5}{2} \times 8.3 = 2.5 \times 8.3 = 20.75 \, \mathrm{J\,mol^{-1}\,K^{-1}}$

✅ **Final Answer:**

$\boxed{20.75 \, \mathrm{J\,mol^{-1}\,K^{-1}}}$

Correct option: B

Question19

The fundamental frequency of transverse wave of a stretched string subjected to a tension T_1 is 300 Hz . If the length of the string is

doubled and subjected to a tension of T_2, the fundamental frequency of the transverse wave in the string becomes 100 Hz , then $T_2 : T_1 =$

(Linear density of the string is constant)

Options:

A.

$1 : 2$

B.

$3 : 4$

C.

$2 : 3$

D.

$4 : 9$

Answer: D

Solution:

The initial fundamental frequency

$$f_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}}$$

$$300 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}} \qquad \ldots(i)$$

The new fundamental frequency

$$f_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{\mu}}$$

$$100 = \frac{1}{2(2L_1)}\sqrt{\frac{T_2}{\mu}}$$

$$100 = \frac{1}{4L_1}\sqrt{\frac{T_2}{\mu}} \qquad \ldots(ii)$$

Divide the Eq. (i) by the Eq. (ii)

$$\frac{300}{100} = \frac{\frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}}}{\frac{1}{4L_1}\sqrt{\frac{T_2}{\mu}}}$$

$$3 = \frac{\frac{1}{2}\sqrt{T_1}}{\frac{1}{4}\sqrt{T_2}} \Rightarrow 3 = \frac{4}{2}\sqrt{\frac{T_1}{T_2}}$$

$$3 = 2\sqrt{\frac{T_1}{T_2}} = \frac{3}{2} = \sqrt{\frac{T_1}{T_2}} = \frac{9}{4} = \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = \frac{4}{9}$$

The ratio of the new tension to the initial tension is $\frac{4}{9}$.

Question20

Two sound waves each of intensity I are superimposed. If the phase difference between the waves is $\frac{\pi}{2}$, then the intensity of the resultant wave is

Options:

A.

$2I$

B.

$3I$

C.

$4I$

D.

I

Answer: A

Solution:

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Here, $\phi = \frac{\pi}{2}$ and $I_1 = I_2 = I$

$\therefore \quad I_R = I + I + 2\sqrt{I \cdot I}\cos\frac{\pi}{2} = 2I$

Question21

The angle of a prism made of a material of refractive index $\sqrt{2}$ is 90°. The angle of incidence for a light ray on the first face of the prism such that the light ray suffers total internal reflection at the second face is

Options:

A.

0°

B.

90°

C.

60°

D.

45°

Answer: B

Solution:

$\mu=\sqrt{2}, A=90^{\circ}$

For total internal reflection

$$\begin{aligned} \sin i_C &= \frac{1}{\mu}=\frac{1}{\sqrt{2}}=\sin 45^{\circ} \\ \therefore \quad i_C &= 45^{\circ} \end{aligned}$$

For total internal reflection to occur at the second face, the angle of incidence at the second face (r_2) must be greater than or equal to the critical angle. To ensure total internal reflection, we consider the limiting case where

$r_2=\theta_c=45^{\circ}$

The angle of the prism is related to the angles of refraction at the two faces by

$A=r_1+r_2$

Since $A=90^{\circ}$ and $r_2=45^{\circ}$

$90^{\circ}=r_1+45^{\circ}$
$r_1=90^{\circ}-45^{\circ}=45^{\circ}$

Applying Snell's law at the first face,

$$\mu_{\text{air}} \quad \sin i=\mu_{\text{prism}} \quad \sin r_1$$
$$1 \cdot \sin i=\sqrt{2} \sin 45^{\circ}$$
$$\sin i=\sqrt{2} \cdot \frac{1}{\sqrt{2}}$$
$$=1=\sin 90^{\circ}$$
$$\therefore \quad i=90^{\circ}$$

Question22

The total magnification produced by a compound microscope is 24 when the final image is formed at the least distance of distinct vision. If the focal length of the eyepiece is 5 cm , the magnification produced by the objective is

Options:

A.

4

B.

4.8

C.

120

D.

6

Answer: A

Solution:

When the final image is at the least distance of distinct vision,

$M=M_0 \cdot M_e$

as we know, $M_e=1+\frac{D}{F_e}$

$M_e = 1 + \frac{25}{5} = 6$

$So,\ 24 = M_0 \times 6$
$M_0 = 4$

Question23

In Young's double slit experiment with light of wavelength λ, the intensity of light at a point on the screen where the path difference becomes $\frac{\lambda}{3}$ is (I is intensity of the central bright fringe)

Options:

A.

I

B.

$\frac{1}{2}$

C.

$\frac{1}{3}$

D.

$\frac{I}{4}$

Answer: D

Solution:

Given, $\Delta x = \frac{\lambda}{3}$

$$\theta = \pi \cdot \frac{\Delta x}{\lambda}$$
$$\theta = \frac{\pi \cdot \lambda}{3 \cdot \lambda} = \frac{\pi}{3}$$

So, $I' = I_{\max} \quad \cos^2 \theta$

$$I' = I_{\max} \cdot \cos^2\left(\frac{\pi}{3}\right)$$
$$I' = \frac{I_{\max}}{4} \quad \therefore I_{\max} = I$$
$$I' = \frac{I}{4}$$

Question24

A thin spherical shell of radius R and surface charge density σ is placed in a cube of side $5R$ with their centers coinciding. The electric flux through one face of the cube is $(\varepsilon_0 = \text{Permittivity of free space})$

Options:

A.

$\frac{2\pi R^2\sigma}{3\varepsilon_0}$

B.

$\frac{\pi R^2\sigma}{3\varepsilon_0}$

C.

$\frac{\sigma}{6\varepsilon_0}$

D.

$\frac{\sigma}{4\pi\varepsilon_0 R^2}$

Answer: A

Solution:

Total charge on spherical shell.

$Q = \sigma A = \sigma\left(4\pi R^2\right) = 4\pi R^2\sigma$

According to Gauss's law total electric flux through a closed surface.

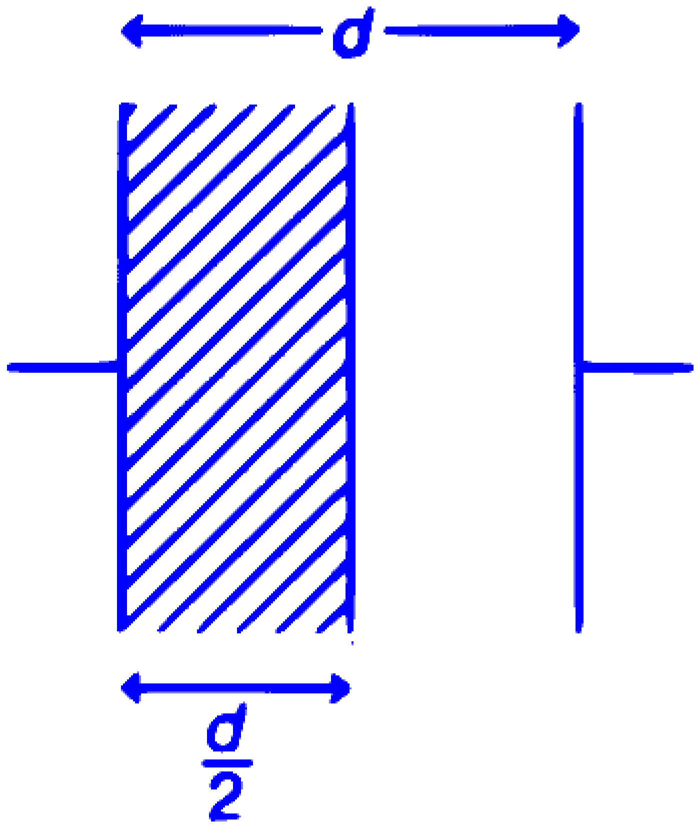
$\phi = \frac{Q}{\varepsilon_0} = \frac{4\pi R^2\sigma}{\varepsilon_0}$

$\therefore$ Flux through one face

$\phi_1 = \frac{\phi}{6} = \frac{4\pi R^2\sigma}{6\varepsilon_0} = \frac{2\pi R^2\sigma}{3\varepsilon_0}$

Question25

As shown in the figure, a dielectric of constant K is placed between the plates of a parallel plate capacitor and is charged to a potential V using a battery. If the dielectric is pulled out after disconnecting the battery from the capacitor, the final potential difference across the plates of the capacitor is



Options:

A.

$(1+\frac{1}{K})2V$

B.

$2KV$

C.

$\frac{2V}{(1+\frac{1}{K})}$

D.

$\frac{V}{2}(1+\frac{1}{K})$

Answer: C

Solution:

When dielectric is filled, we model this as two capacitors in series.

For capacitor with dielectric,

$C_1 = \frac{\varepsilon_0 A}{\frac{d}{2k}} = \frac{2KA\varepsilon_0}{d}$

For capacitor without dielectric,

$C_2 = \frac{\varepsilon_0 A}{\frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$

For series combination,

$$C = \frac{C_1 \times C_2}{C_1 + C_2}$$
$$C = \frac{\frac{2KA\varepsilon_0}{d} \times \frac{2\varepsilon_0 A}{d}}{\frac{2KA\varepsilon_0}{d} + \frac{2\varepsilon_0 A}{d}}$$
$$C = \frac{2\varepsilon_0 A}{d\left(1 + \frac{1}{K}\right)}$$

Charge stored initially,

$Q = CV = \frac{2\varepsilon_0 A}{d(1+\frac{1}{K})} \cdot V$

Now, the entire capacitor just vacuum,

$C' = \frac{\varepsilon_0 A}{d}$

So, final voltage, $V' = \frac{Q}{C'}$

$$= \frac{2\varepsilon_0 AV}{d\left(1 + \frac{1}{K}\right)} \times \frac{d}{\varepsilon_0 A}$$
$$V' = \frac{2V}{\left(1 + \frac{1}{K}\right)}$$

Question26

The drift speed of electrons in a material is found to be $0.3 \mathrm{~ms}^{-1}$ when an electric field of $2 \mathrm{Vm}^{-1}$ is applied across it. The electron mobility (in $\mathrm{m}^2 \mathrm{~V}^{-1} \mathrm{~s}^{-1}$) in the material is

Options:

A.

60×10^{-2}

B.

15×10^{-2}

C.

1350×10^{6}

D.

5400×10^{6}

Answer: B

Solution:

Given

- Drift speed, $v_d = 0.3\ \mathrm{m\,s^{-1}}$
- Electric field, $E = 2\ \mathrm{V\,m^{-1}}$

We are to find **electron mobility** μ, where

$\mu = \frac{v_d}{E}$

Calculation

$\mu = \frac{0.3}{2} = 0.15\ \mathrm{m^2\,V^{-1}\,s^{-1}}$

$\mu = 0.15 = 15 \times 10^{-2}\ \mathrm{m^2\,V^{-1}\,s^{-1}}$

✅ **Answer: Option B**

$\boxed{15 \times 10^{-2}\ \mathrm{m^2\,V^{-1}\,s^{-1}}}$

Question27

The power of an electric motor is 242 W when connected to a 220 V supply. When the motor is operated at 200 V , the current drawn by it is

Options:

A.

1.21 A

B.

1.1 A

C.

1.5 A

D.

1 A

Answer: D

Solution:

Resistance of motor

$R = \frac{V_1^2}{P_1} = \frac{220^2}{242} = 200\Omega$

Current drawn by the motor

$I = \frac{V_2}{R} = \frac{200}{200} = 1 \text{ A}$

Question28

A proton and an alpha particle moving with equal speeds enter normally into a uniform magnetic field. The ratio of times taken by the proton and the alpha particle to make one complete revolution in the magnetic field is

Options:

A.

$1 : \sqrt{2}$

B.

$1 : 2$

C.

$\sqrt{2} : 1$

D.

2 : 1

Answer: B

Solution:

When a charged particle moves in a magnetic field, the time it takes to make one full round (the time period) is given by:

$T = \frac{2\pi m}{Bq}$

Here, m is the mass of the particle, q is its charge, and B is the magnetic field. This means the time period depends on the ratio of mass to charge $(\frac{m}{q})$.

To compare the time periods of a proton (T_p) and an alpha particle (T_α), use this ratio:

$\frac{T_p}{T_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p}$

An alpha particle has a mass four times that of a proton $(m_\alpha = 4m_p)$ and a charge twice that of a proton ($q_\alpha = 2q_p$).

Plug these values in:

$\frac{T_p}{T_\alpha} = \frac{m_p}{4m_p} \times \frac{2q_p}{q_p} = \frac{1}{4} \times 2 = \frac{1}{2}$

This shows that the time taken by a proton is half that of the alpha particle. So, the ratio of their times is

$T_p : T_\alpha = 1 : 2$

Question29

A solenoid of length 50 cm and radius 10 cm has two closely wound layers of windings 100 turns each. If a current of 2.5 A is passing through the windings, the magnetic field (in 10^{-4} T) at a point 5 cm from the axis is

Options:

A.

2π

B.

31.4

C.

4π

D.

zero

Answer: C

Solution:

Number of turns per unit length

$$n=\frac{N}{L}=\frac{200}{0.5}=400 \text{ turns } / \mathrm{m}$$

$$\therefore \quad B=\mu_0 n I$$

$$=4\pi \times 10^{-7} \times 400 \times 2.5$$
$$=4\pi \times 10^{-4} \mathrm{~T}$$

Question30

If the magnetic susceptibility of a substance is 0.6 , then the ratio of permeability of the substance and permeability of free space is

Options:

A.

$6:5$

B.

$7:4$

C.

$8:5$

D.

$3:5$

Answer: C

Solution:

Given:

Magnetic susceptibility $\chi = 0.6$

We need the ratio $\frac{\mu}{\mu_0}$, where

μ = permeability of the substance

μ_0 = permeability of free space

Relation between μ and χ:

$\mu = \mu_0(1+\chi)$

Substitute the value:

$\frac{\mu}{\mu_0} = 1 + \chi = 1 + 0.6 = 1.6$

$\frac{\mu}{\mu_0} = \frac{1.6}{1} = \frac{8}{5}$

✅ **Final Answer:**

$$\boxed{\frac{\mu}{\mu_0} = \frac{8}{5}}$$

Option (C) → 8 : 5

Question31

The plane of a circular coil of resistance 7.5Ω is placed perpendicular to a uniform magnetic field. The flux ϕ (in weber) through the coil varies with time t (in second) as $\phi = 2t^2 + 3t - 2$. The induced power in the coil at time $t = 3$ s is

Options:

A.

7.5 W

B.

15 W

C.

30 W

D.

20 W

Answer: C

Solution:

$\phi = 2t^2 + 3t - 2$

$$\therefore e = \frac{d\phi}{dt} = \frac{d}{dt}\left(2t^2 + 3t - 2\right) = 4t + 3$$
$$\text{at } t = 3\text{ s}, e = 4 \times 3 + 3$$
$$e = 15\text{volt}$$

Induced power, $P = \frac{e^2}{R} = \frac{15^2}{7.5} = 30$ W

Question32

The frequency of an alternating voltage is 50 Hz . The time taken for instantaneous voltage to increase from zero to half of its peak voltage is

Options:

A.

$\frac{1}{800}$ s

B.

$\frac{1}{600}$ s

C.

$\frac{1}{300}$ s

D.

$\frac{1}{200}$ s

Answer: B

Solution:

$$V = V_0 \sin \omega t$$
$$\Rightarrow \frac{V_0}{2} = V_0 \sin \omega t$$
$$\Rightarrow \frac{1}{2} = \sin \omega t \Rightarrow \omega t = \frac{\pi}{6}$$
$$\Rightarrow 2\pi f t = \frac{\pi}{6} \Rightarrow 2\pi \times 50 \times t = \frac{\pi}{6}$$
$$\Rightarrow \quad t = \frac{1}{600} \text{ s}$$

Question33

The dielectric constant of a medium is 8 and its relative permeability is 200 . If an electromagnetic wave of frequency 100 MHz travels in this medium, then its wavelength is

Options:

A.

15 m

B.

15 cm

C.

7.5 m

D.

7.5 cm

Answer: D

Solution:

Velocity of electromagnetic wave in medium

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{200 \times 8}}$$
$$v = 7.5 \times 10^6 \text{ m/s}$$
$$\Rightarrow \quad v\lambda = 7.5 \times 10^6$$
$$\Rightarrow \quad \lambda = \frac{7.5 \times 10^6}{v} = \frac{7.5 \times 10^6}{100 \times 10^6}$$
$$= 0.075 \text{ m} = 7.5 \text{ cm}$$

Question34

Photons of energy 4.5 eV are incident on a photosensitive material of work function 3 eV . The de-Broglie wavelength associated with the photoelectrons emitted with maximum kinetic energy is nearly

Options:

A.

$10\overset{o}{\mathrm{A}}$

B.

$5\overset{o}{\mathrm{A}}$

C.

$20\overset{o}{\mathrm{A}}$

D.

$15\overset{o}{\mathrm{A}}$

Answer: A

Solution:

The maximum kinetic energy

$$\mathrm{KE}_{\max} = E - \phi$$
$$\mathrm{KE}_{\max} = 4.5\mathrm{eV} - 3\mathrm{eV}$$
$$\mathrm{KE}_{\max} = 1.5\mathrm{eV}$$

The de-Broglie wavelength is,

$$\lambda = \frac{12.27}{\sqrt{KE_{\max}}}\overset{o}{\mathrm{A}}$$
$$\lambda = \frac{12.27}{\sqrt{1.5}}\overset{o}{\mathrm{A}} \Rightarrow \lambda \approx \frac{12.27}{1.2247}\overset{o}{\mathrm{A}}$$
$$\lambda \approx 10.018\overset{o}{\mathrm{A}}$$

The de-Broglie wavelength associated with the photoelectrons is approximately $10.02\overset{o}{\mathrm{A}}$.

Question35

If the difference in the frequencies of the first and second lines of Lyman series of hydrogen atom is f, then the difference in frequencies of the first and second lines of Balmer series of hydrogen atom is

Options:

A.

$\frac{3f}{4}$

B.

f

C.

$\frac{7f}{20}$

D.

$\frac{9f}{16}$

Answer: C

Solution:

The frequency for hydrogen spectral lines is found using:

$$f = RC\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Lyman Series (moving to $n_1 = 1$):

First line: Electron goes from $n_2 = 2$ to $n_1 = 1$

$$f_1^{(L)} = RC\left(1 - \frac{1}{4}\right) = RC\frac{3}{4}$$

Second line: Electron goes from $n_2 = 3$ to $n_1 = 1$

$$f_2^{(L)} = RC\left(1 - \frac{1}{9}\right) = RC\frac{8}{9}$$

Difference between the two Lyman frequencies:

$$
\begin{aligned}
f &= f_2^{(L)} - f_1^{(L)} \\
&= RC\left(\frac{8}{9} - \frac{3}{4}\right) \\
&= RC\left(\frac{32-27}{36}\right) = RC\frac{5}{36}
\end{aligned}
$$

Balmer Series (moving to $n_1 = 2$):

First line: Electron goes from $n_2 = 3$ to $n_1 = 2$

$$f_1^{(B)} = RC\left(\tfrac{1}{4} - \tfrac{1}{9}\right) = RC\tfrac{5}{36}$$

Second line: Electron goes from $n_2 = 4$ to $n_1 = 2$

$$f_2^{(B)} = RC\left(\tfrac{1}{4} - \tfrac{1}{16}\right) = RC\tfrac{3}{16}$$

Difference between the two Balmer frequencies:

$$
\begin{aligned}
f' &= f_2^{(B)} - f_1^{(B)} \\
&= RC\left(\frac{3}{16} - \frac{5}{36}\right) \\
&= RC\left(\frac{27-20}{144}\right) \\
&= RC\left(\frac{7}{144}\right)
\end{aligned}
$$

Relationship between differences:

$$
\begin{aligned}
\frac{f'}{f} &= \frac{\left(\frac{7}{144}\right)}{\left(\frac{5}{36}\right)} = \frac{7}{144} \times \frac{36}{5} = \frac{252}{720} = \frac{7}{20} \\
f' &= \frac{7}{20} f
\end{aligned}
$$

Question36

The average energy of a neutron produced in the fission of ${}^{235}_{92}\mathrm{U}$ is

Options:

A.

160×10^{-13} J

B.

320×10^{-15} J

C.

320×10^{-13} J

D.

160×10^{-15} J

Answer: B

Solution:

Average energy of a neutron emitted in ${}^{235}_{92}\mathrm{U}$ fission is 2 MeV

$$\begin{aligned} \therefore \quad \varepsilon &= 2\mathrm{MeV} \\ &= 2 \times 10^6 \times 1.6 \times 10^{-19} \mathrm{~J} \\ &= 3.2 \times 10^{-13} \mathrm{~J} = 320 \times 10^{-15} \mathrm{~J} \end{aligned}$$

Question37

If 96.875% of a radioactive substance decays in 10 days, then the half life of the substance is (in days)

Options:

A.

10

B.

5

C.

4

D.

2

Answer: D

Solution:

We use this formula to find how much of the substance is left after a certain time:

$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

It is said that 96.875% of the substance has decayed after 10 days.

This means that only 3.125% of the substance is left, because $100\% - 96.875\% = 3.125\%$.

We can also write 3.125% as a fraction: $3.125\% = \frac{1}{32}$.

So, $\frac{N}{N_0} = \frac{1}{32}$. We set up the formula:
$\left(\frac{1}{2}\right)^{t/T_{1/2}} = \frac{1}{32}$
Now, $\frac{1}{32} = \left(\frac{1}{2}\right)^5$, so $t/T_{1/2} = 5$.

So, $t = 5 \times T_{1/2}$.
We know $t = 10$ days. So,
$T_{1/2} = \frac{10}{5} = 2$ days
The half-life of the substance is 2 days.

Question38

The power gain and voltage gain of a transistor connected in common emitter configuration are 1800 and 60 respectively. If the change in the emitter current is 0.62 mA , then the change in the collector current is

Options:

A.

0.60 mA

B.

0.58 mA

C.

0.52 mA

D.

0.48 mA

Answer: A

Solution:

Power gain $=$ Current $\times$ Voltage gain

$\Rightarrow 1800 =$ Current gain $(\beta) \times 60$

$\Rightarrow \quad \beta = \frac{1800}{60} \Rightarrow \beta = 30$

$\Rightarrow \quad \frac{\Delta I_C}{\Delta I_B} = 30 \Rightarrow \frac{\Delta I_B - \Delta I_B}{\Delta I_B} = 30$

$\Rightarrow \quad \frac{\Delta I_E}{\Delta I_B} - 1 = 30$

$\Rightarrow \quad \frac{0.62}{\Delta I_B} = 31$

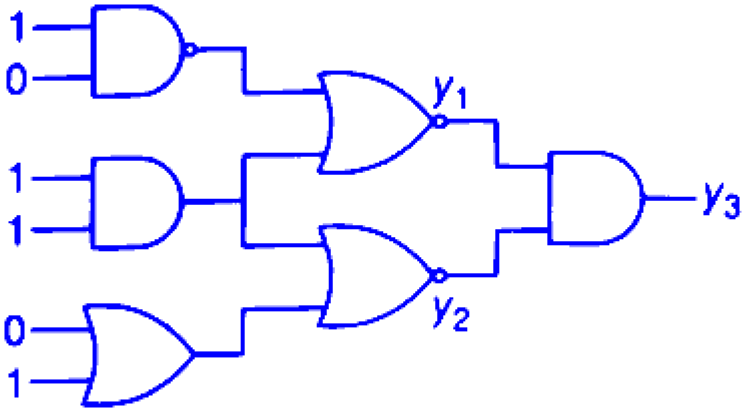
$\Rightarrow \quad \Delta I_B = \frac{0.62}{31} = 0.02 \text{ mA}$

$\therefore \quad \Delta I_C = \Delta I_E - \Delta I_B = 0.62 - 0.02$

$= 0.60 \text{ mA}$

Question39

Six logic gates are connected as shown in the figure. The values of y_1, y_2 and y_3 respectively are



Options:

A.

$(0, 1, 0)$

B.

$(1, 0, 0)$

C.

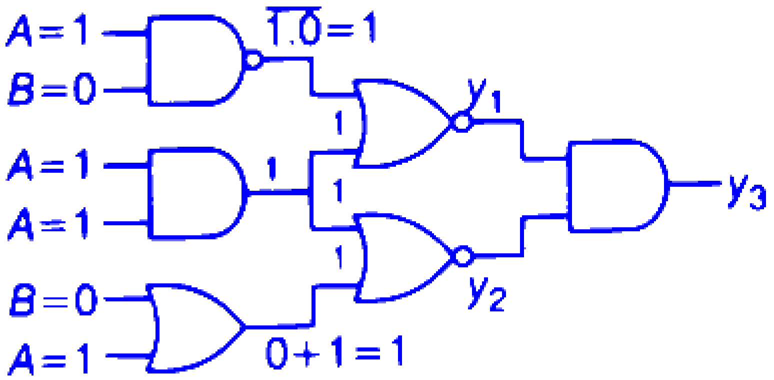
$(0, 0, 1)$

D.

$(0, 0, 0)$

Answer: D

Solution:



$y_1 = \overline{1+1} = \overline{1} = 0$
$y_2 = \overline{1+1} = \overline{1} = 0$
$y_3 = y_1 \cdot y_2 = 0.0 = 0$

Question40

For commercial telephonic communication, the frequency range adequate for speech signals is

Options:

A.

$20 \text{ Hz} - 20\text{kHz}$

B.

$300 \text{ Hz} - 3100 \text{ Hz}$

C.

$200\text{MHz} - 600\text{MHz}$

D.

$300\text{kHz} - 8000\text{kHz}$

Answer: B

Solution:

The correct answer is:

✅ **Option B:** $300\,\mathrm{Hz} - 3100\,\mathrm{Hz}$

Explanation:

Human speech contains frequency components roughly between **100 Hz and 8 kHz**, but for **commercial telephony**, only the most important frequencies for intelligibility are transmitted.

- The **telephone system** limits the transmitted audio bandwidth to approximately **300 Hz – 3400 Hz**, though sometimes it's cited as **300 Hz – 3100 Hz**, depending on standards and equipment.
- This limited range saves bandwidth while still allowing speech to sound natural and be easily understood.

So, the frequency range adequate for **commercial telephonic communication** is approximately **300 Hz to 3100 Hz (or 3400 Hz)**.
